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A Discrete Mathematical Model of the Variable State of the Pandemic

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Abstract—**The paper describes and analyzes a discrete mathematical model of the variable state of the pandemic, which is important for determining production quantities of vaccines and antiviral drugs, predicting the number of infected persons, and the intensity of the process of disseminating information or new ideas to the public. According to the system of differential equations of the pandemic, a discrete mathematical model in vector-matrix form was developed and the equilibrium of the model in the space state was proved. As a result of the implementation of the pandemic model, the discrete dynamic curves of the variable state were obtained in a Matlab package.**

***Keywords-** pandemic; mathematical modeling Covid-19; open systems; equation of the discrete variable state, superposition of epidemic waves, prediction.*

I. INTRODUCTION

The COVID-19 pandemic is primarily the most serious social problem throughout the world, due to the structure and dynamics of contact networks, people's daily lives and actions - at home, in transport, at work, in shopping malls and so on. It is just impossible to detect all contacts. However, we can indirectly reconstruct them through modeling, based on data characteristic of human behavior in different situations. In order to succeed in containing the spread of infection, it is necessary to analyze the dynamics of the spread of infection, calculate the burden on the health system, and make realistic forecasts. All of that can be achieved through the adequate mathematical model.

Mathematical modeling of the COVID-19 pandemic has started since the moment of the first outbreak of the disease in China and is going on intensively to this day. Different models are used for this purpose, which take into account different confirming factors. The more complex the model, the more unknown parameters it contains and the more difficult is to estimate this model [1][2].

Real systems are open and heterogeneous, due to which, from time to time, they create new sources of infection, forming a chain of infection transmission from infected people. This suggests that static data collected mostly from the pandemic curves represent a superposition of many local waves of the pandemic.

In addition, when modeling, we must keep in mind that statistical data have have a large error margin. Therefore, we are dealing with an incorrectly stated problem and, consequently, not with a single solution.

Based on the above analysis, it is necessary to develop a discrete mathematical model of the space state that is adequate to the reality and can be used not only to describe and predict the total number of patients, but also the number of deaths and the number of recovered people.

II. MAIN PART

The equation system of the variable state of the pandemic can be represented in the vector-matrix form [3][6][7][8]:

$$\dot{\mathbf{X}} = \mathbf{A}x + \mathbf{B}u \,, \qquad (1)$$

where

$$\mathbf{A} = \begin{bmatrix} -\alpha & -\beta & 0 \\ \beta & -\gamma & 0 \\ \alpha & \gamma & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix};$$

α , β , γ - are the positive constants of proportionality, which measure the intensity of reducing and completing the appropriate groups.

Information flow in the model is illustrated by ths structural scheme (Figure. 1).

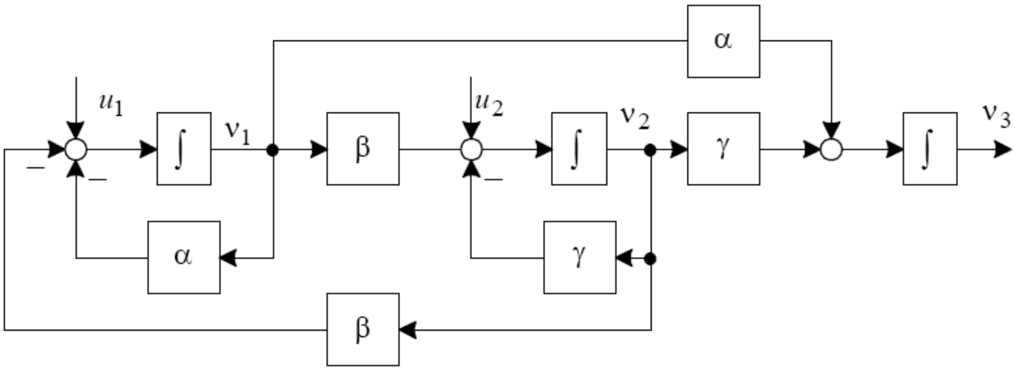

Figure. 1. Structural scheme of the pandemic

Assume that speed of the emergence of a new state in the equation (1) is equal to zero or $u_1 = 0$ the speed of the emergence of newly infected is determined as $u_2(0) = 1$ da $u_2(k) = 0$, when $k \geq 1$. This indicates that only one infected is emerged at a start time, which is equivalent to a pulse action at the point of entry. Constant time of the characterizing equation $S^2 + 2S + 2 = 0$ of (1) is $1/\xi\omega_n = 1$ sec, thus we choose $T = 0{,}2$ sec. We should note that practically time can be measured in months, while the input influence in thousand people. Let us write the equation of the state (1) in discrete form [4]:

$$x(k+1) = \begin{bmatrix} 0{,}8 & -0{,}2 & 0 \\ 0{,}2 & 0{,}8 & 0 \\ 0{,}2 & 0{,}2 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0{,}2 \\ 0 \end{bmatrix} u_2(k) \qquad (2)$$

The system's repsonse at a start time $t = T$, that is, when $t = 2T$ and $x_1(0) = x_2(0) = x_3(0)$ is equal to the initial conditions:

$$x(1) = \begin{bmatrix} 0 \\ 0{,}2 \\ 0 \end{bmatrix} \qquad (3)$$

From here on $u_2(k)$, when $k \geq 1$ will be equal to zero and at a time $t = 2T$, the system's repsonse is determined as follows

$$x(2) = \begin{bmatrix} 0{,}8 & -0{,}2 & 0 \\ 0{,}2 & 0{,}8 & 0 \\ 0{,}2 & 0{,}2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0{,}2 \\ 0 \end{bmatrix} = \begin{bmatrix} -0{,}04 \\ 0{,}16 \\ 0{,}04 \end{bmatrix} \qquad (4)$$

Analogously, at a time $t = 3T$, we shall obtain:

$$x(3) = \begin{bmatrix} -0{,}064 \\ 0{,}120 \\ 0{,}064 \end{bmatrix} \qquad (5)$$

The following values are also simply calculated. The solution to the equation (2) in the case of $T = 0{,}2$ sec step is shown in Figure 2. Claculations were made using a Matlab 9.0 software package [4][5][9][11].

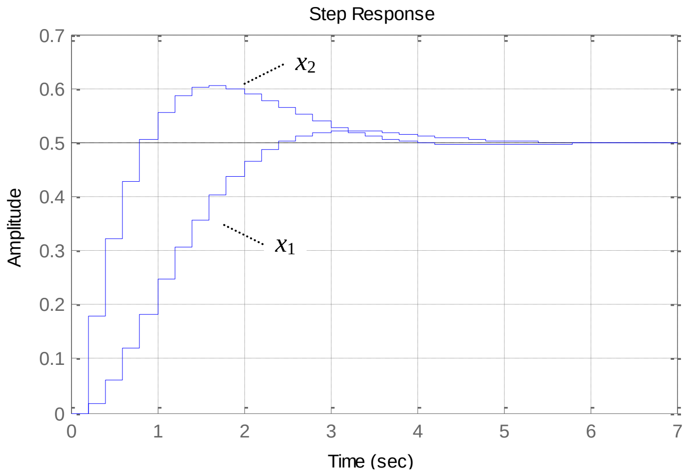

Figure. 2. The discrete dynamic characteristics of the pandemic.

It should be noted that the value x_1 cannot be negative, and in this case, it is obtained due to the inadequacy of the model.

The incidence of epidemic diseases is described more precisely by the system of non-linear equations [3]:

$$\dot{x}_1 = -\alpha x_1 - \beta x_1 x_2 + u_1(t)$$
$$\dot{x}_2 = \beta x_1 x_2 - \gamma x_2 + u_2(t) \qquad (6)$$
$$\dot{x}_1 = -\alpha x_1 + \gamma x_2$$
,

where the relation between the population groups is represented by the nonlinear term $x_1 x_2$. Analogously to the analog system, let us assume that $\alpha = \gamma = \beta = 1$, $u_1(t) = 0$, $u_2(t) = 1$ and $u_2(k) = 0$, when $k \geq 1$[3].

When calculating the time chracteristic of a non-linear system, the discretization method of the equation of state [ლის გაწნ]. We choose the discretization interval $T = 0{,}2$ sec and set the initial condition $X^T(0) = [100]$. We put $t = kT$ into equation (6) and we shall obtain:

$$\frac{x_1(k+1) - x_1(k)}{T} = -x_1(k) - x_1(k)x_2(k)$$
$$\frac{x_2(k+1) - x_2(k)}{T} = -x_1(k)x_2(k) - x_2(k) + u_2(k)$$
$$\frac{x_3(k+1) - x_3(k)}{T} = x_1(k) + x_2(k) \qquad (7)$$

Let us calculate $x_i(k+1)$ from (7) and take into consideration that $T = 0{,}2$ sec, and we shall obtain:

$$x_1(k+1) = 0{,}8x_1(k) - 0{,}2x_1(k)x_2(k)$$
$$x_2(k+1) = 0{,}8x_2(k) + 0{,}2x_1(k)x_2(k) + 0{,}2u_2(k)$$
$$x_3(k+1) = x_3(k) + 0{,}2x_1(k) + 0{,}2x_2(k)$$

Then at a sart time, when $t = T$, we shall obtain:

$$x_1(1) = 0{,}8;\ x_2(1) = 0{,}2;\ x_3(1) = 0{,}2$$

Using again (8) and considering that $u_2(t) = 0$, we shall obtain:

$$x_1(2) = 0{,}608;\ x_2(2) = 0{,}192;\ x_3(2) = 0{,}40.$$

Analogously, when, $t = 3T$ we get:

$$x_1(3) = 0{,}463;\ x_2(3) = 0{,}177;\ x_3(3) = 0{,}56$$

Based on the analysis of the linear and non-linear systems, it is obvious that the response of a non-linear system differs significantly from the response of a linear system, and it adequately describes the real state.

III. Conclusion

Based on the analysis, it was found that the system of nonlinear equations of the pandemic adequately describes the real state.

The developed model allows to predict the number of infected people as well as the intensity of the dissemination of information or new ideas to the public.

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