Missing Values Estimation Comparison in Split- Plot Design

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Abstract---- **The present study focuses on treating the missing values in the split- plot design. Three methods have been used to treat the missing values: Coons, Haseman and Gaylor, and Rubin method. To make preference among these methods some statistical measurements have been used, which are absolute error (AE), mean squares error (MSE) and Akaike information criterion (AIC). From the practical work it is concluded that: In the case of one missing value was obtained the same estimates for missing value. As in cases of two and three missing values show that the best method for estimating missing values is Coons method.**

Keywords-- ***Split- Plot Design, Estimating Missing Values, Mean Squares Error,*** *Akaike Information Criterion.*

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I. INTRODUCTION

In designed experiments sometimes it is so happens that the observations on some experimental units are not available. For example in an industrial experiment, the observations are misplaced or cannot be collected; in a medical experiment, patients may with draw from the treatment programmer or the experimenter may fail to record the results. Similarly in an agricultural experiment the plants may be eaten away are animals.

In all situations the resulting data are called non- orthogonal [7]. One of the first papers on the subject of estimating the missing yield was published by Allan and Wishart (1930) [10]. Yates (1933) showed that by choosing values that minimize the residual sum of squares, one can obtain the correct least squares estimates of all estimable parameters as well as the correct residual sum of squares[1][3].Bartlett (1937), Anderson (1946) and Coons (1957) has used the analysis of covariance model to analyze the experiments with missing data[9].

Rubin (1972) used non- iterative technique to estimate missing values and in a way that using least squares and make the sum of squares error equal to zero.

Haseman and Gaylor (1973) described a simple non- iterative technique to estimate missing values by solving a set of simulations linear equations that can be written directly.

Recently Subramani and Ponnuswamy (1989) have discussed the non-iterative least squares estimation of missing values in experimental designs and presented randomized block designs and latin square designs[7]. Bhatra (2013) studied the estimation of m missing observations by specifying the positions and by not positions of the missing values are presented in case of a randomize block design [1].

Three methods have been used to treat the missing values: Coons, Haseman and Gaylor, and Rubin method. To make preference among these methods some statistical measurements have been used, which are absolute error (AE) mean square error (MSE) and Akaike information criterion (AIC).

II. SPLIT- PLOT DESIGN

Split –plot designs were originally developed by Fisher (1925) for use in agricultural experiments [5].The split -plot design usually used because of some limitation in space or to facilitate treatment application. The two factors are divided into a main plot effect and a sub- plot effect. The precision is greater of the sub- plot factor than it is for the main- plot factor. If one factor is more important to the researcher, and if the experiment can facilitate it, then the sub- plot factor should be used for this factor.

The mathematical model for split- plot design is [11]:

$$y_{ijk} = \mu + \alpha_i + \rho_k + \beta_j + (\alpha\beta)_{ij} + \delta_{ik} + \varepsilon_{ijk} \qquad (1)$$

$i = 1,2, \ldots, a$
$j = 1,2, \ldots, b$
$k = 1,2, \ldots, r$

Where:

y_{ijk} : The value of any observation
μ: General mean
α_i : Effect of main- plot factor (A)
ρ_k: Effect of block
β_j: Effect of sub- plot factor (B)
$(\alpha\beta)_{ij}$: Effect of the interaction between A and B
δ_{ik} : Error of main plot
ε_{ijk}: Error of sub plot

The analysis of variance for split- plot design is:

Sum square of total:

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r} Y_{ijk}^2 - \frac{Y_{...}^2}{abr} \quad (2)$$

Sum square of block:

$$SSblock = \frac{\sum Y_{..k}^2}{ab} - \frac{Y_{...}^2}{abr} \quad (3)$$

Sum square of factor A (main- plot):

$$SSA = \frac{\sum Y_{i..}^2}{br} - \frac{Y_{...}^2}{abr} \quad (4)$$

Sum square of error A:

$$SSE_{(a)} = \frac{\sum Y_{i.k}^2}{b} - \frac{Y_{...}^2}{abr} - SSA - SSblock \quad (5)$$

Sum square of factor B (sub- plot):

$$SSB = \frac{\sum Y_{.j..}^2}{ar} - \frac{Y_{...}^2}{abr} \quad (6)$$

Sum square of interaction effect AB:

$$SSAB = \frac{\sum Y_{ij.}^2}{r} - \frac{Y_{...}^2}{abr} - SSA - SSB \quad (7)$$

Sum square of error B:

$$SSE_{(b)} = SST - SSblock - SSA - SSB - SSAB - SSE_{(a)} \quad (8)$$

Each has an associated degree of freedom. Mean squares are defined as sums of squares divided by degrees of freedom, the analysis of variance as shown in table(1).

TABLE1: ANOVA FOR SPLIT- PLOT DESIGN

(S.O.V.)	(d.f.)	(S.S.)	(M.S.)	(F.Cal)
Blocks	r-1	$SSblock$	$\frac{SSblock}{r-1}$	$F_{cal} = \frac{MSblock}{MSE(a)}$
A	a-1	SSA	$\frac{SSA}{a-1}$	$F_{cal} = \frac{MSA}{MSE(a)}$
Error(a)	(a-1)(r-1)	$SSE_{(a)}$	$\frac{SSE_{(a)}}{(a-1)(r-1)}$	
B	b-1	SSB	$\frac{SSB}{b-1}$	$F_{cal} = \frac{MSB}{MSE(b)}$
AB	(a-1)(b-1)	$SSAB.$	$\frac{SSAB}{(a-1)(b-1)}$	$F_{cal} = \frac{MSAB}{MSE(b)}$
$Error(b)$	a(b-1)(r-1)	$SSE_{(b)}$	$\frac{SSE_{(b)}}{a(b-1)(r-1}$	
$Total$	abr-1	SST		

III. METHODS OF ESTIMATING MISSING VALUES

A. Coons Method

Coons (1957) was used analysis of covariance model to analyze the experiments with missing values .the technique employs the computational procedures of a covariance analysis using a dummying X covariance as follows:
In the case of one missing value:
To estimate of missing value by covariance analysis conducting the following steps [9], [5]:

1) Consider the original data as the dependent variable y of the covariance analysis and inset the value of zero in the cell which has the missing observation.
2) Define a variable x where:
$X = 0 \quad if \quad Y \neq 0$
$X = -n \quad if \quad Y = 0$
Where: n is the total number of observation in the experiment including the missing value.
3) Carry out the analysis of covariance.
4) Compute the estimate of the regression coefficient:
$$\hat{\beta}_E = \frac{E_{XY}}{E_{XX}} \quad (9)$$
And multiply by n to estimate the missing value:
$$X = n\hat{\beta}_E \quad (10)$$

In the case of more than one missing value:

1) Put $Y = 0$ for all missing values.
2) Define a variable X_m where:
$X_m = 0 \quad iff \quad Y \neq 0$
$X_m = -n \quad iff \quad Y = 0$
3) With more than one missing observation a multiple covariance analysis is required.

The computations required to obtain the sum of products $\sum X_m X_n$ and $\sum X_m Y$, since each X_m is associated with a single missing value and therefore has only one non-zero cell.
In computing $\sum X_m X_n$, two situations may be encountered.

a) The two missing values associated with X_m and X_n occur in the same level of the given source of variation.
$\sum X_m X_n = n$(Degree of freedom for the given source of variation)

b) The two missing values occur in the different levels of the given source of variation.
$\sum X_m X_n = -n$(Degree of freedom for the given source of variation)

4) Compute the estimates of the regression coefficients ($\hat{\beta}_{1E}$, $\hat{\beta}_{2E}$, …, $\hat{\beta}_{mE}$) by solving m equations:

$$\left.\begin{array}{c} E_{X_1X_1}\hat{\beta}_{1E} + E_{X_1X_2}\hat{\beta}_{2E} + \cdots + E_{X_1X_m}\hat{\beta}_{mE} = E_{X_1Y} \\ \vdots \\ E_{X_mX_1}\hat{\beta}_{1E} + E_{X_mX_2}\hat{\beta}_{2E} + \cdots + E_{X_mX_m}\hat{\beta}_{mE} = E_{X_mY} \end{array}\right\} \quad (11)$$

We estimate the missing values by the following formula:

$$X_i = n\hat{\beta}_{iE} \quad , i = 1,2,3, \dots, m \qquad (12)$$

B. Haseman and Gaylar Method

Haseman and Gaylor (1973) suggested a non- iterative technique to estimate m missing values by solving m of simulations linear equations, the formula as follows[6]:

$$(r-1)(b-1)Y_h + \sum_{g\neq h}^{m} Y_g\left[\psi_{gh}(A_3) - r\psi_{gh}(A_1) - b\psi_{gh}(A_2)\right] = rT_h(A_1) + bT_h(A_2) - T_h(A_3) \qquad (13)$$

Where:

$r = Number\ of\ replicates$
$b = Number\ of\ levels\ of\ the\ second\ factor.$

$$\psi_{gh}(A_1) = \begin{cases} 1, & If\ Y_h\ and\ Y_g\ are\ of\ different\ levels\ of\ factor\ B, but\ from\ the\ same\ levels\ of\ factor\ A\ and\ in\ the\ same\ block. \\ 0, & \text{Otherwise.} \end{cases}$$

$$\psi_{gh}(A_2) = \begin{cases} 1, If\ Y_h\ and\ Y_g\ are\ of \text{ a particular level for the factors A and B.} \\ 0, Otherwise. \end{cases}$$

$$\psi_{gh}(A_3) = \begin{cases} 1, If\ Y_h\ and\ Y_g\ are\ of\ the\ same\ level\ for\ factor\ A. \\ 0, Otherwise. \end{cases}$$

$T_h(A_1) =$ Total for main unit containing the missing value.

$T_h(A_2) =$ Total of all sub units that receive the treatment combination$(a_i b_j)$.

$T_h(A_3) =$ Total of all observations that receive the ith level of A.

C. Rubin Method

In (1972) Rubin used non- iterative technique to estimate missing values and in a way that using least squares and make the sum of squares error equal to zero [2].

$$X = -PR^{-1} \qquad (14)$$

Where:
$P, X =$ Vector $(1 \times m)$.
$R = Non-singular\ matrix\ (m \times m).$

$$e_{ijk} = Y_{ijk} - \frac{Y_{ij.}}{b} - \frac{Y_{.jk}}{r} + \frac{Y_{.j.}}{br} \qquad (15)$$

$$r_{kk} = 1 - \frac{1}{b} - \frac{1}{r} + \frac{1}{br} \qquad (16)$$

$$r'_{kk} = \frac{1}{br} \qquad (17)$$

IV. STATISTICAL MEASUREMENTS

After estimating missing values, the missing values are replaced by the estimated values and the usual computations procedures of the analysis of variance is applied to the augmented data set with some modifications subtract one from the error degree of freedom for each missing value.
Some statistical measurements have been used, which are: mean squares error (MSE) is calculated as shown in equation (8) and table (1).
And absolute error (AE) is the absolute of the difference between estimated value and real value, and calculated as follows:

$$AE = |y_i - \hat{y}_i| \qquad (18)$$

Where:
y_i: Real value.
$\hat{y}_i$: Estimated value.

And Akaike information criterion (AIC) is a measure of the relative quality of statistical methods for a given set of data, is calculated as follows:

$$AIC = n \ln \sigma^2 + 2(k+1) \qquad (19)$$

Where:

σ^2 : Mean square of error.

k : Number of variables in the model.

n : Total number of observations.

V. THE PRACTICAL PART

A. Data Description

Data on height (cm) of eucalyptus plants from a field trial under split- plot design with two treatments, three blocks given in (Jayaraman). Let A denoted the main- plot factor (pit size) and B, the sub plot factor (fertilizer treatments), then the resulting data is as follows [8]:

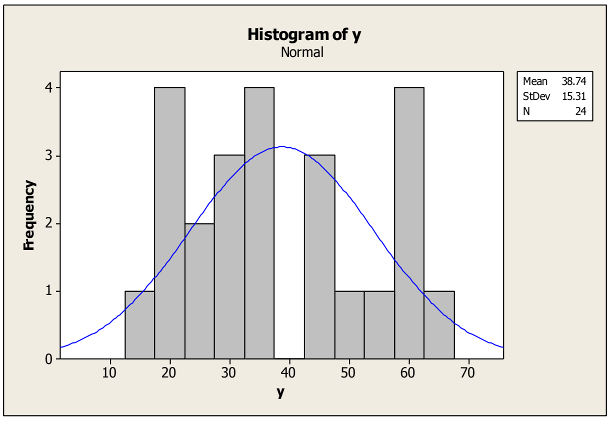
Missing values in the experiment are not missing originally, but I assumed it was missing.

TABLE 2: DATA EXPERIMENT

A	B	Blocks I	Blocks II	Blocks III	Total
a_1	b_1	25.38	61.35	37.00	123.73
	b_2	46.56	66.73	28.00	141.29
	b_3	66.22	35.70	35.7	137.62
	b_4	30.68	58.96	21.58	111.22
Total		168.84	222.74	122.28	513.86
a_2	b_1	19.26	55.8	57.6	132.66
	b_2	19.96	33.96	31.7	85.62
	b_3	22.22	58.4	51.96	132.6
	b_4	16.82	45.6	26.55	88.97
Total		78.26	193.76	167.83	439.85
		247.1	416.5	290.11	953.71

Before analyzing the data, should be verified from the distribution of the data. To test the normal property was used the histogram, as shown in figure (1) as follows.

FIGURE 1: HISTOGRAM FOR DATA



The above chart in figure (1), explained that the data experiment distributed normal distribution, and to test the homogeneity the hypothesis is given by:

$$\left.\begin{array}{l} H_0: all\ variances\ are\ equal \\ H_1: at\ least\ one\ variance\ not\ equale \end{array}\right\} \qquad (20)$$

The value of Bartlett's test equal to (2.57) with p- value (0.92), for test the p- value is greater than the value of level of significant ($\alpha = 0.05$), this means cannot reject the null hypothesis and there is no problem of homogeneity of variance.

Carry out the analysis of variance which is given in table (3):

TABLE 3: ANOVA FOR DATA EXPERERIMENT.

(S.O.V.)	(d.f.)	(S.S.)	(M.S.)	(F.Cal)	(F.Tab.) $\alpha = 0.05$
Blocks	2	1938.5	969.25		
A	1	228.35	228.35	0.39	4.75
Error(a)	2	1161.34	580.67		
B	3	487.82	162.61	1.01	3.49
AB	3	388.21	129.40	0.81	3.49
Error(b)	12	1928.15	160.68		
Total	23	6132.37			

Three methods have been used to treat the missing values: Coons, Haseman and Gaylor, and Rubin method. To make preference among these methods some statistical measurements have been used, which are absolute error (AE), mean squares error (MSE) and Akaike information criterion (AIC).

B. Estimating Missing Value

Case 1: One Missing Value

In the table (2) assume that y_{122} is the observation. Let X_1 be the corresponding observed value but is unknown. Now we estimate the missing value X_1 based on the following methods:

1. Coons Method

Let n equal the total number of observations in the experiment including the missing one. Consider the original data as the depended variable Y of the covariance analysis and insert the value of zero in the cell which has the missing observation. Set up a concomitant variable X which takes the value of –n in the cell corresponding to the substituted zero value and the value of zero elsewhere.

TABLE 4: ANCOVA FOR CACE 1.

S.O.V	d.f	$\sum XY$	$\sum X^2$
Block A E(a)	2 1 2	-162.33 -7.28 120.53	48 24 48
B AB E(b)	3 3 12	240.26 200.64 638.28	72 72 288
Total	23		552

By using equation (9), we get

$$\hat{\beta}_E = \frac{E_{XY}}{E_{XX}} = 2.216$$

A missing value is estimated by equation (10):

$$X = 24(2.216) = 53.19$$

2. Haseman and Gaylar Method

By equation (13), we get

$$(3-1)(4-1)X = 3(156.01) + 4(74.56) - 447.13$$
$$X = 53.19$$

3. Rubin Method

By equation (16) and (17) we get

$$P = 0 - \frac{156.01}{4} - \frac{74.56}{3} + \frac{447.13}{12} = -26.595$$
$$\boldsymbol{r} = 1 - \frac{1}{4} - \frac{1}{3} + \frac{1}{12} = 0.5$$

By equation (14), we get

$$X = \frac{-(-26.595)}{0.5} = 53.19$$

The usual analysis of variance is calculated as show in table (5).

TABLE 5: THE ANALYSIS AFTER ESTIMATING MISSING VALUE

Methods	Estimate of missing value	AE	MSE	AIC
Coons	53.19	8.16	166.95	132.82
H&G	53.19	8.16	166.95	132.82
Rubin	53.19	8.16	166.95	132.82

By using the above mentioned methods the missing values were estimated to compression among these methods by relating on AE, MSE and AIC of the estimated value in order to ascertain its proximity to the real value.

For one missing value all of the three methods indicated above produce the same result.

Case 2: Two Missing Values

That y_{122} and y_{213} are the observations. Let X_1 and X_2 be the corresponding observed values but are unknown. Now we estimate the missing values X_1 and X_2 based on the following methods:

1. Coons Method

A multiple covariance analysis is used to handle the problem of two missing values. Assign the value of zero to the two missing values ($y_{122}= 0$) and ($y_{213}= 0$). Set up two concomitant variables X_1 and X_2 for each missing values. Each of $X_1 = 0$ in all cells except in that cell corresponding to , in that one cell $X_1 = -n$. Similarly, each of $X_2 = 0$ in all $X_2 = -n$. The computation of a multiple covariance is given in table (6).

TABLE 6: ANCOVA WITH TWO MISSING VALUES

S.O.V	d.f	$\sum X_1Y$	$\sum X_2Y$	$\sum X_1X_2$	$\sum X_m^2$
Block A E(a)	2 1 2	-219.93 -64.88 178.13	131.84 64.88 -28.73	-24 -24 24	48 24 48
B AB E(b)	3 3 12	188.66 109.12 638.28	34.22 129.8 497.36	-24 24 0	72 72 288
Total	23	829.38	829.38	-24	552

By using equation (11), we get

$$288\,\hat{\beta}_{1E} + 0\hat{\beta}_{2E} = 638.28$$
$$0\,\hat{\beta}_{1E} + 288\,\hat{\beta}_{2E} = 497.36$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = \begin{pmatrix}288 & 0\\ 0 & 288\end{pmatrix}^{-1}\begin{pmatrix}638.28\\ 497.36\end{pmatrix}$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = \begin{pmatrix}2.215\\ 1.726\end{pmatrix}$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\end{pmatrix} = \begin{pmatrix}2.215\\ 1.726\end{pmatrix}$$

Missing values are estimated by equation (12):

$$\begin{pmatrix}X_1\\ X_2\end{pmatrix} = \begin{pmatrix}53.16\\ 41.424\end{pmatrix}$$

2. Haseman and Gaylar Method

By equation (13), we get

$$6X_1 + X_2 = 319.14$$
$$X_1 + 6X_2 = 248.68$$

$$\begin{pmatrix}X_1\\ X_2\end{pmatrix} = \begin{pmatrix}6 & 1\\ 1 & 6\end{pmatrix}^{-1}\begin{pmatrix}319.14\\ 248.68\end{pmatrix}$$

$$\begin{pmatrix}X_1\\ X_2\end{pmatrix} = \begin{pmatrix}47.61\\ 33.51\end{pmatrix}$$

3. Rubin Method

By equation (16) and (17), we get

$$p_1 = 0 - \frac{156.01}{4} - \frac{74.56}{3} + \frac{447.13}{12} = -26.595$$
$$p_2 = 0 - \frac{110.23}{4} - \frac{75.06}{3} + \frac{382.25}{12} = -20.72$$

$$\boldsymbol{r_{kk}} = 1 - \frac{1}{4} - \frac{1}{3} + \frac{1}{12} = 0.5$$
$$\boldsymbol{r'_{kk}} = \frac{1}{12} = 0.083$$

$$p = (-26.595 \quad -20.72)$$

$$R = \begin{pmatrix}0.5 & 0.083\\ 0.083 & 0.5\end{pmatrix}$$

Missing values are estimated by equation (14):

$$\begin{pmatrix}X_1\\ X_2\end{pmatrix} = \begin{pmatrix}47.62\\ 33.53\end{pmatrix}$$

TABLE 7: THE ANALYSIS AFTER ESTIMATING TWO MISSING VALUES

Methods	Estimate of missing value	AE	MSE	AIC
Coons	53.16 41.42	8.19 16.18	170.59	133.340
H&G	47.61 33.51	13.74 24.09	175.29	133.995
Rubin	47.62 33.53	13.73 24.07	175.27	133.992

The analysis in table (7) showed that the (MSE) and (AIC) of the Coons method less than the (MSE) and (AIC) of the H&G and the Rubin method, and estimated values of Coons method are closer to the real values.

Case 3: Three Missing Values

Assume that y_{122} , y_{213} and y_{231} are the observations. Let X_1 , X_2 and X_3be the corresponding observed values but are unknown.

We estimate the missing values X_1 , X_2 and X_3 based on the following methods:

1. Coons Method

A multiple covariance analysis is used to handle the problem of two missing values. Assign the value of zero to the two missing values (y_{122}= 0) , (y_{213}= 0) and (y_{231}). Set up three concomitant variables X_1 , $X_2$and X_3 for each missing values. The computation of a multiple covariance is given in table (8).

TABLE 8: ANCOVA WITH THREE MISSING VALUES

S.O.V	$\sum X_1Y$	$\sum X_2Y$	$\sum X_3Y$	$\sum X_1X_2$	$\sum X_1X_3$	$\sum X_2X_3$	$\sum X_m^2$
Block A E(a)	242.15 -87.1 200.35	109.63 87.1 -50.95	132.52 87.1 251.3	-24 -24 24	-24 -24 24	-24 -24 24	48 24 48
B AB E(b)	166.44 131.34 638.28	12 107.58 541.8	-184.84 21.86 499.22	-24 24 0	-24 24 0	-24 24 0	72 72 288
Total	807.16	807.16	807.16	-24	-24	-24	552

By using equation (11):

$$288\,\hat{\beta}_{1E} + 0 + 0 = 638.28$$
$$0 + 288\,\hat{\beta}_{2E} + 0 = 541.8$$
$$0 + 0 + 288\,\hat{\beta}_{3E} = 499.22$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\\ \hat{\beta}_3\end{pmatrix} = \begin{pmatrix}288 & 0 & 0\\ 0 & 288 & 0\\ 0 & 0 & 288\end{pmatrix}^{-1}\begin{pmatrix}638.28\\ 541.8\\ 499.22\end{pmatrix}$$

$$\begin{pmatrix}\hat{\beta}_1\\ \hat{\beta}_2\\ \hat{\beta}_3\end{pmatrix} = \begin{pmatrix}2.216\\ 1.881\\ 1.733\end{pmatrix}$$

Missing values are estimated by equation (12):

$$\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix} = \begin{pmatrix}53.19\\ 45.15\\ 41.60\end{pmatrix}$$

2. Haseman and Gaylar Method

By equation (13), we get

$$6X_1 + X_2 + X_3 = 319.14$$
$$X_1 + 6X_2 + X_3 = 270.90$$
$$X_1 + X_2 + 6X_3 = 249.61$$

$$\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix} = \begin{pmatrix}6 & 1 & 1\\ 1 & 6 & 1\\ 1 & 1 & 6\end{pmatrix}^{-1}\begin{pmatrix}319.14\\ 270.90\\ 249.61\end{pmatrix}$$

$$\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix} = \begin{pmatrix}42.84\\ 33.19\\ 28.93\end{pmatrix}$$

3. Rubin Method

By equation (16) and (17), we get

$$p_1 = 0 - \frac{156.01}{4} - \frac{74.56}{3} + \frac{447.13}{12} = -26.595$$
$$p_2 = 0 - \frac{110.23}{4} - \frac{75.06}{3} + \frac{360.03}{12} = -22.575$$

$$p_3 = 0 - \frac{56.04}{4} - \frac{110.38}{3} + \frac{360.03}{12} = -20.8$$

$$\boldsymbol{r_{kk}} = 1 - \frac{1}{4} - \frac{1}{3} + \frac{1}{12} = 0.5$$

$$\boldsymbol{r'_{kk}} = \frac{1}{12} = 0.083$$

$$p = (-26.595 \quad -22.575 \quad -20.8)$$

$$R = \begin{pmatrix}0.5 & 0.083 & 0.083\\ & 0.5 & 0.083\\ & & 0.5\end{pmatrix}$$

Missing values are estimated by equation (14):

$$\begin{pmatrix}X_1\\ X_2\\ X_3\end{pmatrix} = \begin{pmatrix}42.87\\ 33.23\\ 28.97\end{pmatrix}$$

TABLE 9: THE ANALYSIS AFTER ESTIMATING THREE MISSING VALUES

Methods	Estimates of missing values	AE	MSE	AIC
Coons	53.19 45.15 41.60	8.16 12.45 19.38	185.67	135.375
H&G	42.84 33.19 28.93	18.51 24.41 6.71	191.46	136.112
Rubin	42.87 33.23 28.97	18.48 24.37 6.75	191.36	136.099

The analysis in table (9) showed that the (MSE) and (AIC) of the Coons method less than the (MSE) and (AIC) of the H&G and the Rubin method, and estimated values of Coons method are closer to the real values.

VI. CONCLUSIONS

The results of the study of estimating missing values are summarized and tabulated in tables (5, 7, and 9)) which contain the MSE, AE and AIC, we have observed that:

1. In the case of one missing value was obtained the same estimates for missing value.

2. The results of application in cases of two and three missing values show that the best method for estimating missing values is Coons method, because it is minimum mean squares error, minimum absolute mean square error and minimum Akiakes information criterion**.**

3. Increase the number of missing values leads to increased difference between estimated values given by different methods.

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