Ultrasonic Signal Detection With Sequential Equivalent Time Sampling Based on Compressed Sensing

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Abstract—**This paper presents an ultrasonic signal detection approach using sequential equivalent time sampling (SETS) based on compressed sensing (CS). SETS has been incorporated in digital acquisition system to capture a repetitive waveform with a single analog to digital converter (ADC) clocked at a rate that is much lower than the signal's Nyquist rate. The proposed approach considers recovering repetitive ultrasonic signal with high time resolution from samples taken by SETS. The CS measurement matrix is constructed in the context of SETS technique. A basis function is constructed to realize the ultrasonic signal sparse representation, which paves the way for applying CS theory to ultrasonic signal sub-Nyquist rate sampling. Experimental results indicate that, the proposed sampling model is feasible, and the ultrasonic signal could be reconstructed from a small number of SETS samples. Compared to the traditional SETS time-alignment signal reconstruction, the number used to signal reconstruction could be significantly reduced.**

Keywords- equivalent time sampling; ultrasonic signal detection; compressed sensing; orthogonal matching pursuit

I. INTRODUCTION

Ultrasonic imaging technique are well established in medical diagnostics [1], flaw detection [2], and imaging in the context of large engineering structures. The traditional approach in sampling ultrasonic signal is governed by Shannon-Nyquist sampling theorem. In order to achieve highly accurate detection, a very high time resolution is required. Signal should be sampled by an analog-to-digital converter (ADC) clocked at a high sampling rate. A single ADC solution based on uniform sampling becomes nonviable and sometimes unavailable.

In instrumentation application [3]-[5], the sequential equivalent time sampling (SETS) provides an architectural solution to sampling repetitive signal with a single ADC that is clocked at sub-Nyquist sampling rate. For repetitively generated ultrasonic signal, SETS could be used improve the time resolution at a low cost. In SETS system, signal is reconstructed from multiple acquisitions. Compared to real time sampling technique, SETS often takes considerably longer sampling time to produce a reconstructed waveform with high time resolution. To achieve a desired time resolution, one has to sequentially capture sample. The number of acquisition runs is proportional to the time resolution. In order to reduce this unwanted time delay, in this paper, a signal reconstruction algorithm in the framework of compressed sensing (CS) is proposed.

CS [6], [7] is a new signal processing theory, which can reconstruct a high equivalent sampling rate signal from a small number of low speed samples. The feasibility of applying CS to sample spectrally sparse periodic signals has been demonstrated [8], [9].

Sparse representation is a fundamental premise in the use of CS theory. However, not all signals have concise sparse expression in the conventional basis, such as Fourier basis. Special case is the signal consisting of stream of short pulses, which are prevalent in applications: ultrasonic detection, UWB communications, and radar systems. In many ultrasonic detection applications, signals are repetitively generated, and SETS technique can be used to capture this family of signals. In order to use CS algorithm, the ultrasonic signal need to be sparsely represented. In many ultrasonic signal applications, we can characterize it by the time-delays and weighted amplitudes, which indicate the position and strength of the various scatterers, respectively. Since the degrees of freedom of underlying signal are decided by the number of scatterers rather than the maximum frequency presented in the signal, signal is sparse. In this paper, we consider incorporate CS theory into SETS sampling technique to improve its efficient. The well-known Whittaker-Shannon interpolation formula is used to construct the CS measurement matrix.

In the remaining of this paper, we briefly review the SETS sampling method in Section II. CS background is introduced, and a sparse representing method for ultrasonic signal is presented in Section III. CS based SETS signal reconstruction algorithm is presented in Section IV. The simulation results are reported in Section V. Further discussions are summarized in Section VI.

II. SEQUENTIAL EQUIVALENT TIME SAMPLING

Equivalent time sampling can be divided into two types: random equivalent time sampling (RETS) and sequential equivalent time sampling (SETS). Both techniques could sample high speed signal with a single low speed ADC clocked at a sub-Nyquist rate, and signal could be successfully reconstructed from multiple acquisitions. In this paper, our focus is SETS.

The basic principle of SETS sampling is illustrated in Fig. 1. In SETS, the test signal should be periodic or repetitively generated. SETS system captures one sample at each acquisition. When trigger signal is detected, the first samples is taken after an fixed time delay T_d. At the next acquisition, after trigger signal, the second sample is taken at $T_d + \Delta T$ (ΔT is very short and well defined delay). At the m^{th} acquisition, the m^{th} sample is taken at $T_d + m\Delta T$. Until the signal with the desired time solution could be successfully reconstructed, this process is repeated. After enough samples are collected, the signal is reconstructed by time aligning the samples.

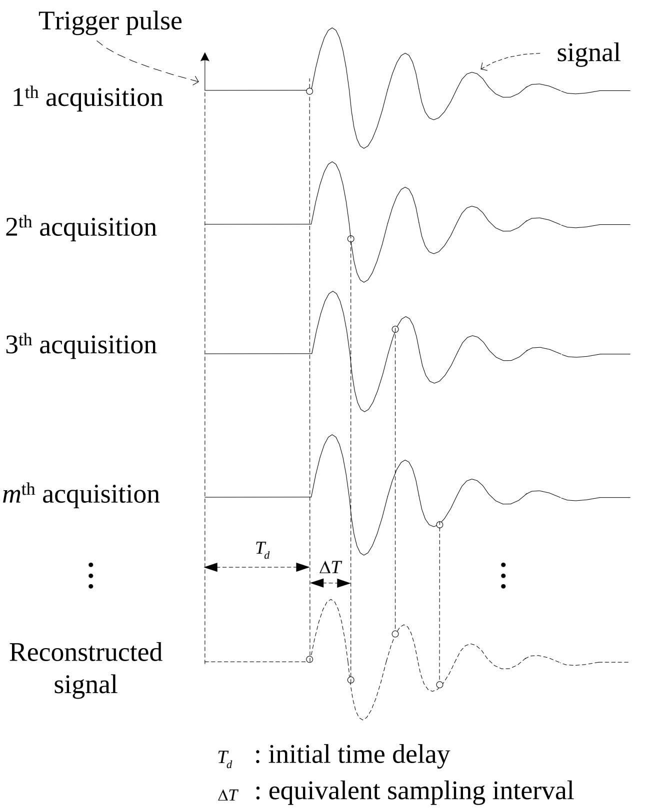

Fig. 1. SETS sampling process.

In the composited signal, the time interval between adjacent samples is ΔT, the equivalent sampling rate of reconstructed signal is $1/\Delta T$. On the other hand, the smallest sampling interval of ADC is T_d. Generally, $T_d >> \Delta T$, sub-Nyquist sampling is achieved. For a reconstructed signal supported in $[0, T]$, there are

$$N = \frac{T}{\Delta T} \tag{1}$$

samples need to be captured. Obviously, SETS is inefficient.

In order to improve sampling efficiency of SETS, we make some changes in the sampling stage of SETS. We only capture P SETS samples, and $P << N$. After trigger signal, the p^{th} sample is taken at $T_d + T_{rand}$, and T_{rand} is defined as

$$T_{rand} = T_d + m_p \Delta T \tag{2}$$

where $1 \le p \le P$, and $0 \le m_p \le N - 1$. The sampling process is depicted in Fig. 2.

Since $P << N$, a small number of samples are captured, the efficiency of SETS is improved. Note from Fig. 2, there are many samples are unavailable, signal cannot be reconstructed using time alignment method. In this paper, we present a CS-based algorithm to reconstruct signal from the un-enough SETS samples.

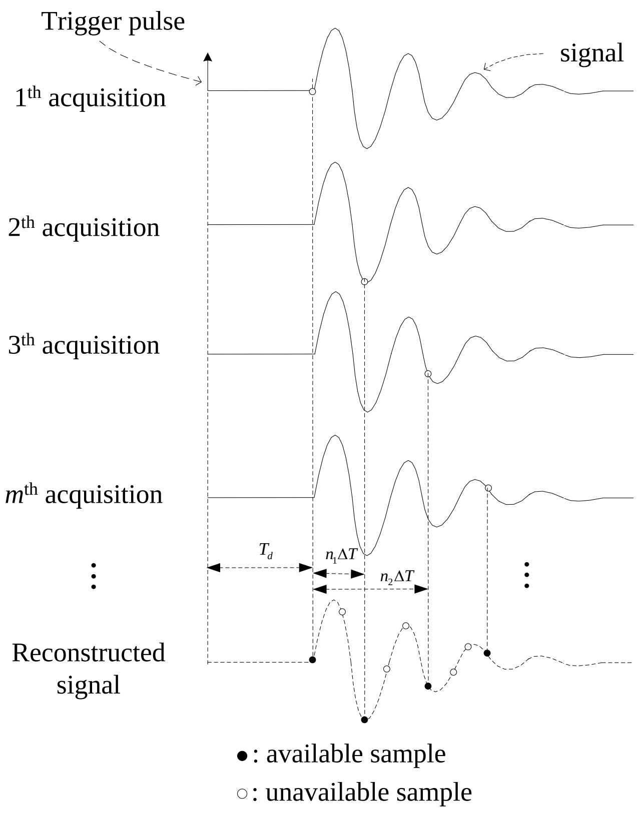

Fig. 2. Random SETS sampling process.

III. ULTRASONIC SIGNAL SPARSE REPRESENTATION

In this section, we review the CS theory background and present a sparse representation approach for ultrasonic signal.

A. Compressed Sensing Background

CS is an emerging data sampling paradigm that has received much attention. The promise of CS is that a sparse or compressible signal can be recovered from a small set of projections. To make this possible, there are two conditions: sparsity, which pertains to the signals of interest, and incoherence, which pertains to the sensing modality.

Sparsity expresses the idea that the "information rate" of a continuous time signal may be much smaller than suggested by its bandwidth presented in signal, or that a discrete-time signal depends on a number of degrees of freedom that is comparably much smaller than its (finite) length. More precisely, CS exploits the fact that many natural signals are sparse or compressible in the sense that they have concise representations when expressed in the proper basis, such as Fourier basis, wavelet basis, etc. The basis can be selected according to the signal's peculiarities.

In CS, the signal to be reconstructed is denoted by an N dimensional vector $\mathbf{x}$. In the practical application, $\mathbf{x}$ would be the unknown analog signal sampled at the equivalent sampling rate f_e. Using SETS or other methods, a set of *measurements* of the elements of $\mathbf{x}$ is obtained and denoted by a vector $\mathbf{y}$. In SETS, each element of $\mathbf{y}$ (P dimensional vector) is the output of a low-rate ADC. It can be represented as a weighted linear combination of elements in $\mathbf{x}$ that contains evenly spaced samples of the unknown waveform, with the sampling rate $f_e > f_{Nyquist}$ ($f_{Nyquist}$ is the Nyquist rate of the signal to be measured). Let these weights be arranged in a *measurement matrix* $\mathbf{\Phi}$, one may express the relation between $\mathbf{x}$ and $\mathbf{y}$ as follows:

$$\mathbf{y}=\mathbf{\Phi x} \tag{3}$$

In SETS, signal $\mathbf{x}$ is assumed to be periodic, it has a discrete Fourier spectrum. Such a signal is *spectrally sparse* if only very few Fourier coefficients have significant magnitudes while other are nearly zero. In other words, the energy of the signal is concentrated on few spectral coefficients. In particular, a *K-sparse* signal $\mathbf{x}$ has K significant spectral coefficients where K is the *sparsity level*. If $\mathbf{x}$ is sampled from a *K-sparse* periodic analog signal $x(t)$, it can be approximated by a linear combination of K ($K << N$) discrete Fourier basis functions, i.e.,

$$\boldsymbol{\alpha} = \sum_{i=1}^{N} \psi_i \mathbf{x} = \sum_{k=1}^{K} \psi_{n_k} \mathbf{x} \tag{4}$$

where $\psi_{n_k} \in \mathbf{\Psi}$, $n_k \in \{1, 2, \ldots, N\}$, and $\mathbf{\Psi}$ is the inverse discrete Fourier transform (IDFT) matrix. Let $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_N]^T$ be the coefficients vector of $\mathbf{x}$ in $\mathbf{\Psi}$, and $\boldsymbol{\alpha}$ is a sparse vector consisting of K non-zero Fourier coefficients. (3) can be re-expressed as:

$$\mathbf{y}=\mathbf{\Phi\Psi x} \tag{5}$$

In practice, most of natural signals are sparse or near sparse, and they can be recovered from their compressible samples. Obviously, the degree of freedom of $\mathbf{y}$ is less than that of $\mathbf{x}$, and (5) has no unique solution. However, the sparsity of signal $\mathbf{x}$ guarantees that the original signal can be precisely recovered by using l_0 norm optimization problem:

$$\min \|\boldsymbol{\alpha}\|_0 \quad s.t. \quad \mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi\Psi}\boldsymbol{\alpha}\,. \tag{6}$$

where the l_0 norm $\|\cdot\|_0$ counts the number of nonzero entries in a vector.

B. Ultrasonic Signal Sparse representation

In ultrasonic application, an ultrasonic signal is transmitted. After reflected by the objects, signal reached at the receiver with a time delay t_d. Through analyzing the received signal, one can locate the positions of objects. Fig. 3 depicts an ultrasonic testing application. To locate the object distance with high precision, one needs to digitize the received signal with high sampling rate. And the precision is inverse proportional to the sampling interval.

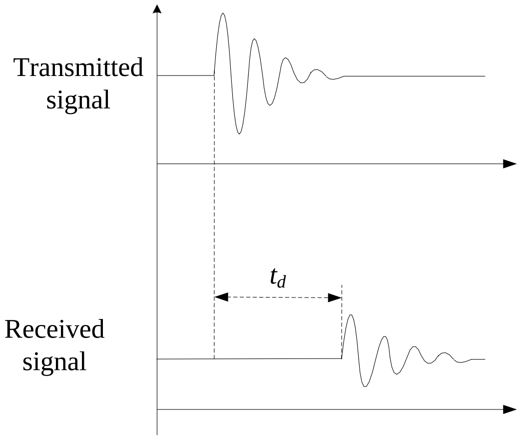

Fig. 3. Basic principle of ultrasonic pulse echo technique.

As described in Section II-A, sparsity is the fundamental premise in CS theory application. However, ultrasonic signal cannot be sparsely expressed in the conventional basis. In order to apply CS theory, we need to construct sparse basis function for ultrasonic signal. As shown in Fig. 3, the received signal are weighted versions of the transmitted one with different time-delays, and the received signal can be expressed as follows:

$$x(t) = \sum_{k=1}^{K} \alpha_k s(t - n_k \Delta T) \tag{7}$$

where $s(t)$ is the transmitted pulse with known shape, $\{\alpha_k, n_k\Delta T \mid k = 1, 2, \ldots, K\}$ are unknown amplitudes and time-delays, K is the number of active pulses in the received signal, and $n_k \in [0, N-1]$, and N is the size of signal $\mathbf{x}$ that is to be recovered. $s(t)$ can be arbitrary as long as

$$s(t - n_k \Delta T) = 0,\ \forall n_k \notin [0, N-1],\ k = 1, 2, \ldots, K. \tag{8}$$

In this signal model, there are only K degrees of freedom. $x(t)$ can be treated as sparse signal with sparsity level of K.

Based on the above analysis, we can construct the sparse representation dictionary (basis) as follows:

$$\mathbf{\Psi} := \{\psi_n(t) \mid \psi_n(t) = s(t - n\Delta T)\} \tag{9}$$

where $n \in \{1, 2, \ldots, N\}$.

IV. CS Based Sub-Nyquist Rate Sampling

The ultrasonic signal could be sparsely represented using a specific basis function. In order to reconstruct signal from SETS samples based on CS algorithm. The measurement matrix of SETS system needs to be constructed.

A. *Measurement Matrix*

SETS system starts to take samples after an initial delay T_d, we can shift the samples to the origin in the signal reconstruction stage. Unlike general CS approaches where random measurement matrices are used, the measurement matrix for SETS sampled signals is motivated by the low-rate ADC mechanism. This measurement matrix is formed by the well-known Whittaker-Shannon interpolation formula. According to the Shannon sampling theorem, the observation **y** and the original signal x satisfy the following formula

$$y(p) = \sum_{n=1}^{N} x(nT_e)\,\mathrm{sinc}\left(m_p - n\right) \tag{10}$$

where $1 \le p \le P$, P is the number of SETS samples ($M << N$), $0 \le m_p \le N - 1$, and T_e is the equivalent sampling period of reconstructed signal. In SETS, $T_e = \Delta T$. In general, it requires no more than $c{\cdot}K{\cdot}\log(N)$ [6] random measurements (c is a constant, say 5 in practice) to recover the signal with high probability.

The $(p, n)^{\mathrm{th}}$ element of the measurement matrix $\mathbf{\Phi}$ is as follows:

$$\mathbf{\Phi}(p,n) = \sin c\left(m_p - n\right). \tag{11}$$

According to the interpolation matrix (11), we can establish the relations between the un-enough SETS samples and the uniform equivalent sampling signal of size N that is to be recovered. However, for the periodic signal with infinite length, the above expression only considers one period contribution. We notice that the function sinc(z) damps slowly as the order of $1/z$ and is not compact-supported, and therefore the contributions from other periods cannot be ignored.

Based on the above analysis, (12) can be revised as follow:

$$\begin{aligned}\mathbf{\Phi}(p,n) &= \sum_{i=-\infty}^{+\infty} \sin c\left(m_p - n + iN\right) \\ &\approx \sum_{i=-L/2+1}^{L/2} \sin c\left(m_p - n + iN\right)\end{aligned}. \tag{12}$$

Numerically, if we truncate the summation with L terms, the recovery error is inversely proportional to L. However, the summation itself converges very slowly and a large number of terms should be used to obtaining the perfect reconstruction. To accelerate the summation, the Poisson summation formula is employed:

$$\sum_{k=-\infty}^{+\infty} f(t + kT) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} \hat{f}\left(\frac{k}{T}\right)\exp\left(2\pi j\frac{k}{T}t\right). \tag{13}$$

where $\hat{f}$ is the continuous Fourier transform of f. Because the Fourier transform of $\sin c(x)$ is the rectangular pulse with finite support, from (13), (12) can be rewritten as

$$\begin{aligned}\mathbf{\Phi}(p,n) &= \sum_{i=-\infty}^{+\infty} \sin c\left(m_p - n + iN\right) \\ &= \frac{1}{N}\sum_{i=-N/2+1}^{N/2} \exp\left[2\pi j\frac{i}{N}\left(m_p - n\right)\right]\end{aligned}. \tag{14}$$

With the help of the Poisson summation formula, we transform the infinite summation in time domain to the finite summation in the frequency domain. The matrix constructed by (14) achieves very accurate and efficient numerical performances.

B. *Orthogonal Matching Pursuit Algorithm*

To solve the optimization problem of (6), one may either apply convex programming [10] or use a family of greedy pursuit algorithm [11]. For real time RES signal reconstruction, the large computation cost of convex program makes it a less appealing approach. In this work, we adopt the orthogonal matching pursuit algorithm [12] (OMP), a variant of greedy pursuit algorithm to solve for the sparse vector $\boldsymbol{\alpha}$.

For convenience, denote a matrix $\mathbf{D} = \mathbf{\Phi}{\cdot}\mathbf{\Psi}$. Equation (6) implies that the solution sparse vector $\boldsymbol{\alpha}$ should use as few possible the columns of the **D** matrix to approximate the observation **y**. This can be formulated as a *subset selection problem* where a minimum subset of columns of the **D** matrix is chosen to approximate the observation vector **y** in the least square sense. The OMP algorithm successively chooses an additional column of the **D** matrix to reduce the approximation error. Equivalent, the OMP method begins with a tentative solution of $\boldsymbol{\alpha}$ with a single non-zero entry, and gradually adding non-zero entries one by one until the approximation error of **y** meets a pre-determined criterion. More specifically, denote **B** to be a matrix formed by the subset of columns of the **D** matrix whose column indices correspond to non-zero entries of the $\boldsymbol{\alpha}$ vector. Then, the measurement vector **y** may be approximated by its projection onto the subspace spanned by columns of **B**:

$$\hat{\mathbf{y}} = \mathbf{P_B y} = \mathbf{B}\left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{y} = \mathbf{BB}^{\dagger}\mathbf{y} \qquad (15)$$

where $\mathbf{B}^{\dagger}$ is the *Moore-Penrose pseudo-inverse* of the **B** matrix. Note that the residue $\mathbf{r} = \mathbf{y} - \hat{\mathbf{y}}$ is perpendicular to the subspace spanned by **B**:

$$\mathbf{P_B r} = \mathbf{P_B}\left(\mathbf{y} - \hat{\mathbf{y}}\right) = \mathbf{P_B y} - \mathbf{P_B}\left(\mathbf{P_B y}\right) = \mathbf{P_B y} - \mathbf{P_B y} = \mathbf{0}\,. \qquad (16)$$

Here the property of a project matrix $\mathbf{P_B}.\mathbf{P_B} = \mathbf{P_B}$ is used.

Denote **C** to be a matrix whose columns are formed by the set difference of columns of the **D** matrix and those of the **B** matrix. A *greedy* criterion to select one more column of **C** to be moved to **B** is to choose one that has the smallest angle between **r** and itself. Let **c** and **c'** be columns of **C**, then the best choice of **c** must satisfy

$$\left\|\mathbf{P_c r}\right\| = \left\|\mathbf{r}^T\mathbf{c}\right\| / \left\|\mathbf{c}\right\| \geq \left\|\mathbf{r}^T\mathbf{c}'\right\| / \left\|\mathbf{c}'\right\| = \left\|\mathbf{P_{c'} r}\right\| \qquad (17)$$

Once the new **c** is chosen, the **B** matrix and **C** matrix will be updated, and the residue vector can be updated. The algorithm will be stopped when the norm of **r** is smaller than the preset threshold ε. The $\boldsymbol{\alpha}$ vector than has the form of

$$\boldsymbol{\alpha} = \begin{bmatrix} \mathbf{B}^{\dagger}\mathbf{y} \\ \mathbf{0}_{(N-K)\times 1} \end{bmatrix} \qquad (18)$$

V. EXPERIMENTAL RESULTS

In this section, numerical experiments are reported to investigate the proposed CS based SETS ultrasonic signal reconstruction. In all the experiments, the time delays m_p are randomly selected in [0, N–1]. The length of the reconstructed signal is $N = 500$. The equivalent sampling rate is $f_e = 1/T_e = 50$ MHz. The experimental waveform is a ultrasonic signal, which can be sparsely represented by basis function (7). There are 4 ultrasonic pulse in the received signal, and the received signal frequency is smaller than $f_e/2$. The time delay of ultrasonic pulse is selected in [0, $(N-1)\Delta T$].

First, experiment is performed to compare signal reconstruction between the traditional SETS time-alignment approach and the proposed CS-based reconstruction. In this experiment, the test signal is noisy-free. 200 samples are randomly taken using SETS technique. Fig. 4 shows the comparison of reconstructions by time-alignment approach and CS based approach. Since there are too many samples unavailable, the signal to noise ratio (SNR) of time-alignment is only 12 dB. However, the CS reconstruction achieves about 60 dB. Clearly, the feasibility and advantage of proposed approach are demonstrated.

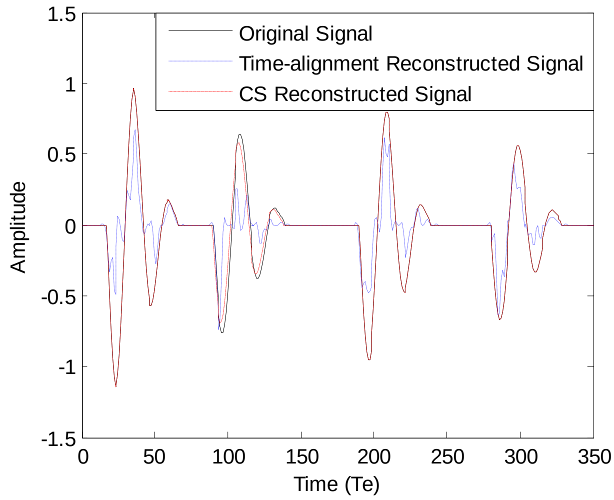

Fig. 4. Comparison of reconstructions.

In the next experiment, the performance with respect to different number of SETS samples is investigated. For the range of 50 to 210 SETS samples in increment of 20 samples, 200 random trials are performed for each specific length. Signal is reconstructed using the proposed CS based algorithm. The averaged SNRs of 500 trials at each length of the random samples is plotted in Fig. 5. Clearly, with the increase of sample sequence, the reconstruction performance improve.

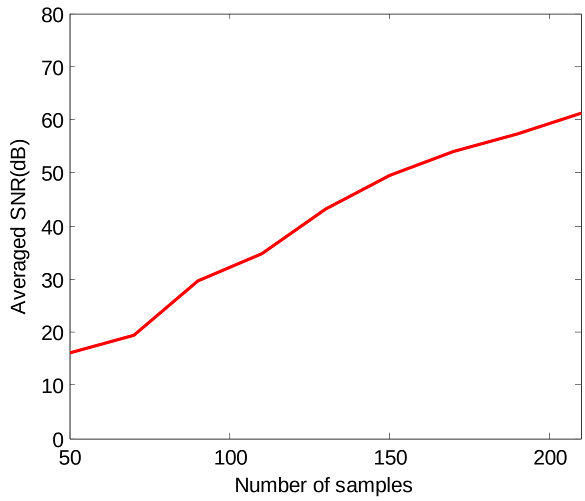

Fig. 5. Signal reconstruction with respect to deferent number of SETS samples.

Finally, the impact of Gaussian distributed noise on the proposed approach is evaluated. 200 SETS samples are used to reconstruct signal, and noise levels over the range of 5 dB to 50 dB in increment of 5dB are tested. 200 trials are repeated for each specific noise level, and the averaged output SNR with respect to different input SNR is plotted in Fig. 6. It is clearly demonstrated that the proposed CS-based SETS approach exhibits robustness against additive Gaussian noise.

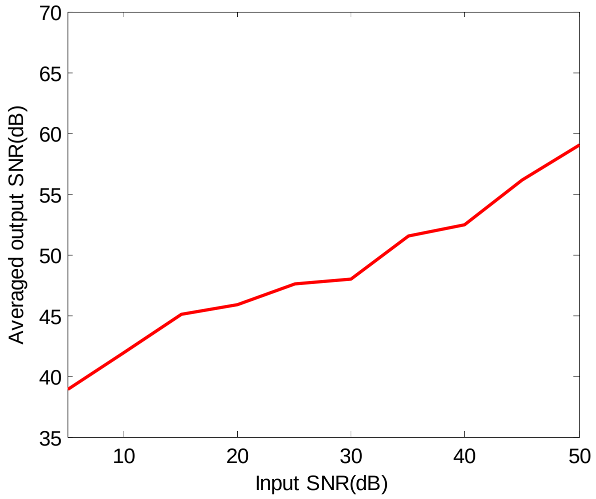

Fig. 6. Reconstructions performance with respect to different noise levels.

VI. CONCLUSION

A CS based compressive detection method is proposed for reconstructing repetitively stimulated ultrasonic waveforms sampled using the SETS technique. The proposed method based on an assumption that the transmitted pulse shape is known. In order to apply CS theory to reconstruct signal, a basis function for ultrasonic signal is introduced, which can be sparsely represent the ultrasonic. The CS measurement matrix specifically for the sequential equivalent time sampling has been constructed by the Shannon sampling theorem and the Poisson summation formula. The numerical experimental results indicate that, with the help of the compressed sensing theory, we can reconstruct the ultrasonic signal from a small number of sequential equivalent time sampling samples, and the proposed reconstruction algorithm is demonstrated to be feasible and effective.

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