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Performance Comparison between Spatial Multiplexing and Non Spatial Multiplexing Systems in Rayleigh Fading Channel with Co-Channel Interference

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Abstract—**This paper investigates and compares the performance between SM and single antenna transmission (non-SM) systems in Rayleigh fading channel limited by one prevailing co-channel interferer, where parameters related to thetransmit power and spectral efficiency are taken into consideration, in order to make a fair comparison between the bothsystems. Monte Carlo simulations were performed for obtaining the results in terms of the mean bit error rate (BER) asa function of per-bit signal-to-noise ratio (E_b/N_0) and signal-to-interference ratio (SIR). The results in this paper can beapplied in order to know the real benefits of the SM technique applied to a cellular system, as WiMAX and LTE.**

Keywords-Cellular Network, Spatial Multiplexing, Co-Channel Interference, BER.

I. INTRODUCTION

New generation of mobile communicationsystem demands more and more broadbandservices. However, the available bandwidthis limited. In such scenario, spatial multiplexing(SM) is a powerful technique usedto increase the transmission data rates withoutbandwidth expansion [1], [2]. This techniquedivides the incoming data into multiple parallel substreams and transmits each on a differentspatial dimension (e.g., a different antenna).

In a cellular network, multiple transmit antennasfor SM are in general collocated at basestation (BS) [3], because high data rates areparticularly interesting for the downlink. Thus,it can be assumed that the number of transmitantennas, N_t, will be larger than the numberof receive antennas, N_r, due to space and costrestrictions on the mobile station (MS).

Another important factor in a cellular networkis that the same channel is reused inspatially separated cells to efficiently utilize thelimited frequency bandwidth. Therefore, theMS's receive antennas suffer from co-channelinterference (CCI) from neighboring cells [2].

Several simulations and analysis for SM systemsin a fading channel have been made [4],[5]. However, in practice, to evaluate the performanceof a SM system in a cellular network,it is also necessary consider the CCI. Someworks that analyze the performance of SM systemsunder fading and co-channel interference,model the CCI using Gaussian approximation[6], [7]. Although in a cellular network could beseveral interfering, generally one of them dominatesthe other [8], such that the co-channelinterference is not Gaussian.

On the other hand, the vast majority of academicand even industrial research about SMsystems (well summarized in [9] and [10]) have showed that using N_t transmit antennas, theoverall data rate compared to a system with anon-SM is enhanced by a factor of N_t withoutrequiring extra bandwidth or extra transmissionpower [1]. However, in these researches areassumed that the both systems use the samemodulation, that is, different spectral efficiencies, ξ. In this way, unfair comparisons betweenSM and non-SM systems are performed.

Based on the idea that SM systems can becompared to non-SM systems that employ highorder modulations, in order that the both systemspresent the same spectral efficiency. Oneneeds to address the following question: Whatis the benefit of a second transmit antenna (SM)at the BS relative to a BS with a non-SM?.Thus, motivated in the last question, in thispaper, we analyze the performance in termsof the mean bit error rate (BER) between SMand non-SM systems applied to a cellular networkdownlink in Rayleigh fading and in thepresence of co-channel interference. In order tomake a fair comparison between both systems,it is presented a methodology for comparison,which takes into consideration the most fundamentalparameter in communication, which aretransmit power and spectral efficiency.

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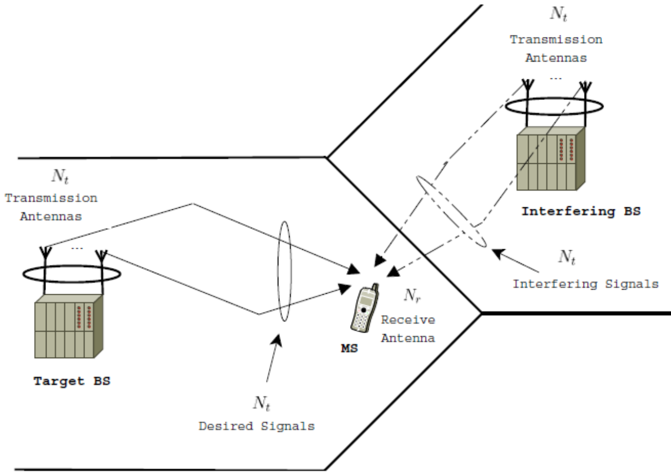

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der of this paper is organized asfollows. In Section II, the system models as well as the comparison methodology are described.Section III presents the details on the meanbit error probability taking into considerationfading and co-channel interference effects. SectionIV describes the numerical and simulationsresults. Finally, the conclusions are made inSection V.

II. SYSTEM DESCRIPTION

We consider a cellular network with SM systemin the presence of a predominant co-channelinterferer as showed in Fig. 1, where the targetand interfering BSs have N_t antennas and theMS has a single antenna. Thus, there will be $N_t$interfering signals, so the received signal at theMS is given by:

$$r(t) = \sum_{i=1}^{N_t} \alpha_i e^{-j\phi_i} A_i\, s_i\, g_{T_s}(t) + \Omega \sum_{i'=1}^{N_t} \alpha_{i'}\, e^{-j\phi_{i'}} A_{i'}\, s_{i'}\, g_{T_s}(t) + n(t). \tag{1}$$

The first term of (1) represents the N_t signalarriving at the MS from the target BS, where$\alpha_i$and$\phi_i$are the channel's fading and phasebetween the i-th transmission antenna and theMS, A_i is the signal amplitude and s_i is the i-thtransmitted symbol, $g_{T_s}(t)$ is the pulse formatwith unitary energy, T_s is the symbol period.The second term of (1) represents the $N_t$signalsarriving at the MS from the co-channel interferingBS, whereΩ is an amplitude factor, whichallow us to vary the signal-to-interference ratio(SIR), $\alpha_{i'}$ and $\phi_{i'}$ are the channel's fading andphase between the i'-th interfering transmissionantenna and the MS, $A_{i'}$ and $s_{i'}$ are the i'-th signal amplitude and interfering transmittedsymbol, respectively, $g_{T_s}(t)$ is the pulse formatwith unitary energy belonging to the interferingBS. Finally, the third term of (1), $n(t)$, is theadditive white Gaussian noise (AWGN) on thereceiver antenna with zero mean and doublesided power spectral density $N_0/2$.

In addition, α_i and $\alpha_{i'}$ are considered tobe independent and identically distributed(i.i.d). Without loss of generality, the signal-to-interferenceratio can be written as the ratiobetween the mean power of the N_t desiredsignals, $\overline{P_0}$, and the mean power of the $N_t$interfering signals, $\overline{P_{CCI}}$, that is:

$$\mathrm{SIR} = \frac{\overline{P_0}}{\overline{P_{CCI}}} = \frac{\sum_{i=1}^{N_t} \overline{\alpha_i^2 A_i^2 s_i^2}}{\Omega^2 \sum_{i'=1}^{N_t} \overline{\alpha_{i'}^2 A_{i'}^2 s_{i'}^2}}, \tag{2}$$

where we considered that $\sum_{i=1}^{N_t} \overline{\alpha_i^2} = \sum_{i'=1}^{N_t} \overline{\alpha_{i'}^2} = N_t\overline{\alpha^2}$ because α_i and $\alpha_{i'}$ are i.i.d Rayleigh randomvariables, and also that $\bar{P} = \sum_{i=1}^{N_t} \overline{A_i^2 s_i^2} = \sum_{i'=1}^{N_t} \overline{A_{i'}^2 s_{i'}^2}$, where $\bar{P}$ is the constellation mean power.

Figure 1. SM system in Rayleigh Fading Channel with One Predominant Co-Channel Interferer.

Therefore, the SIR can be rewritten as:

$$\mathrm{SIR} = \frac{N_t \overline{\alpha^2} \cdot \bar{P}}{\Omega^2 N_t \overline{\alpha^2} \cdot \bar{P}} = \frac{1}{\Omega^2}. \tag{3}$$

Note that for a non-SM system, $N_t = 1$.

A. *Comparison Methodology Proposed*

For a system with a non-SM, the symbol periodis T_s, and the bit rate is given by:

$$R_b = \frac{\log_2 M}{T_s}, \tag{4}$$

Where M is the modulation order. The bandwidthis equal to$B = \frac{1}{T_s}[H_z]$. As a consequencethe spectral efficiency is given by:

$$\xi = \frac{R_b}{B} = \log_2 M \quad [bits/s/H_z]. \tag{5}$$

On the other hand, for the SM system withN_t transmission antennas, the symbol period is$T_s{}' = N_t T_s$, therefore, the bit rate is given by:

$$R_b = \frac{N_t}{T_s{}'} \log_2 M', \tag{6}$$

whereM' is the modulation order for SM system,which must necessarily be lower than themodulation used on the system with a non-SM,in order to maintain the same spectral efficiencyin the both systems. The bandwidth for theSM system is equal to $B = \frac{1}{T_s{}'}[H_z]$and asa consequence the spectral efficiency is givenby:

$$\xi = \frac{R_b}{B} = N_t \log_2 M' [bits/s/H_z]. \tag{7}$$

In order to make a fair comparison, the parameterrelated with the transmission powermust also be taken into consideration. Thus,the total transmitted power for a system witha non-SM is constant and equal to P_t, whereas,the total transmitted power P_t , for a SM systemis equally divided among the transmissionantennas regardless of the number of transmissionantennas, that is, $P_{t_i} = P_t/N_t$, where$P_{t_i}$is the i-th transmitted power for the i-thtransmission antenna.

Thus, based on this methodology, for example,a system sends QPSK symbols on 1transmit antenna, can be compared with a SMsystem sends BPSK symbols on 2 transmit antennas,in this way and used (5) and (7), respectively,the both systems have the same spectralefficiency, which is equal to2 $bits/s/H_z$,therefore, the both systems also present thesame bit rate. With respect to the transmissionpower, the both systems are allocated thesame transmission power, P_t. In the case of SMsystem, each of the 2 transmit antennas hasa transmission power equal to $P_t/2$. Thus, wehave showed a fair comparison between theboth systems considered.

III. PERFORMANCE ANALYSIS

In the presence of one prevailing co-channelinterferer, the instantaneous BER for the i-thtransmission antenna for a SM system withBPSK can be expressed as:

$$P_b\left(\gamma_{b_i},\gamma_{b_{i'}}\right)=\frac{1}{2}Q\left(\sqrt{2\gamma_{b_i}}+\Omega\sqrt{2\gamma_{b_{i'}}}\right)$$
$$+\frac{1}{2}Q\left(\sqrt{2\gamma_{b_i}}-\Omega\sqrt{2\gamma_{b_{i'}}}\right), \qquad (8)$$

where $\gamma_{b_i}=\alpha_i\frac{E_b}{N_0}$ and $\gamma_{b_{i'}}=\alpha_{i'}\frac{E_b}{N_0}$are theinstantaneous SNR for the i-th target and i'-thinterfering transmission antenna, respectively.Note that the target and interfering BSs havethe same number of transmission antennas[2].

In order to obtain the upper bound on themean BER for a SM system in the presenceof one interferer, we need to average over thefading statistics using:

$$\overline{P_b}\leq\frac{1}{N_t}\sum_{i=1}^{N_t}\int_0^\infty\int_0^\infty P_b\left(\gamma_{b_i},\gamma_{b_{i'}}\right)f(\gamma_{b_i})f\left(\gamma_{b_{i'}}\right)d_{\gamma_{b_i}}d_{\gamma_{b_{i'}}}, \qquad (9)$$

where $f(\gamma_{b_i})=\frac{1}{\overline{\gamma_{b\,i}}}e^{\gamma_{b_i}/\overline{\gamma_{b\,i}}}$ and $f\left(\gamma_{b_{i'}}\right)=\frac{1}{\overline{\gamma_{b_{i'}}}}e^{\gamma_{b_{i'}}/\overline{\gamma_{b_{i'}}}}$ are the probability density function(PDF) for γ_{b_i} and $\gamma_{b_{i'}}$, respectively [2]. Unfortunately(9) does not have a closed form andnumerical integration should be used.

This analysis can also be extended to SM systemthat using higher order digital modulationPSK or QAM.

IV. NUMERICAL RESULTS

In this section, we present and compare theperformance between SM and non-SM systemsin the presence of one dominant interferer.In order to make a fair comparison betweenboth systems, the transmit power and spectralefficiency must be taken into considerationas was presented in Section II.A. Monte Carlosimulations were performed for evaluating theperformance of both systems in terms of themean BER as a function of $E_b/N_0$and SIR.We use (N_t,N_r) to denote a link with N_t BStransmit antennas and N_r MS receive antennas.

Fig. 2 shows the mean BER as a function of E_b/N_0 with one dominant co-channel interfererfor a (2, 1) system with BPSK modulation versusa(1, 1) system with QPSK modulation, forSIR = 0, 12, 24, 48 dB. Note that both systemshave the same transmit power and spectralefficiency, which is equal to2 $bits/s/H_z$. In thisfigure, we can see that due to fading and co-channelinterference, there are always BER floorin both systems. For a SIR = 0 dB, we observethat BER floor approaches 1/4 and 3/10 for(2, 1) and (1, 1) systems, respectively. This isdue to the fact that the interference power isequal to the signal power and there is no significantdifference between both systems. ForSIR values in the range of 12 to 24 dB, weobserve an improvement in the performance ofboth systems when compared for a SIR = 0 dB,furthermore, we notice that (1, 1) system withQPSK modulation presents less degradation, interms of the BER, to the effects of fading andco-channel interference in comparison to the(2, 1) system with BPSK modulation, for the SIRvalues into consideration. Whereas, for SIR =48 dB, the interference power in both systemsis very small and can be negligible, so theperformance, in terms of the BER, just considersthe effects of fading. Thus, for example for aBER of 1×10^{-3}, the (1, 1) system has an E_b/N_0 advantage of about 3 dB over the (2, 1) system.Therefore, the (1, 1) system with QPSK modulationis the best choice of implementation because it presents better performance than(2, 1) system with BPSK modulation.

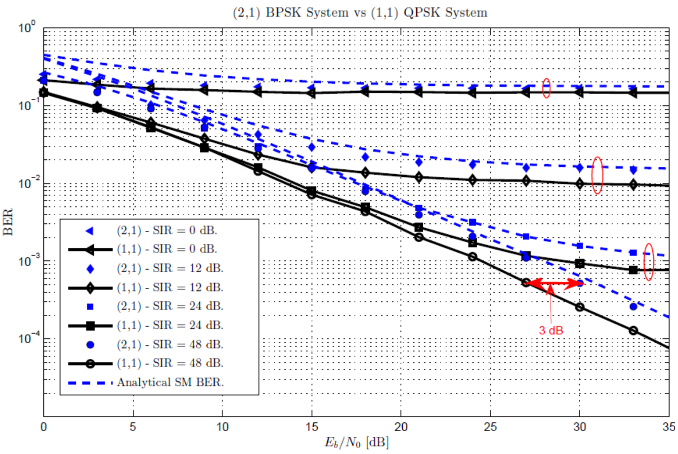

Figure 2. BER as a function of E_b/N_0 and SIR, for(2, 1) system with BPSK and (1, 1) system withQPSK modulation in the presence of one Co-Channel Interferer.

Fig.3 presents performance comparison resultsin the presence of one dominant co-channelinterferer for SIR = 0, 12, 24, 48 dB,between a (3, 1) system with BPSK modulationand a (1, 1) system with 8-PSK modulation,where, both systems have the sametransmit power and spectral efficiency equal to3 $bits/s/H_z$. For SIR = 48 dB, again, the (1, 1)system has a better performance than (3, 1)system, because it provides an advantage ofabout 0.5 dB in terms of E_b/N_0 over the (3, 1)system, making the (1; 1) system the best implementationoption. On the other hand, for SIRvalues in the range of 12 to 24 dB, we observethat the (3, 1) system with BPSK modulationpresents less performance degradation due tothe effects of fading and co-channel interferencein comparison with the (1, 1) system with 8-PSK modulation. This occurs because the 8-PSK modulation is much more susceptible tointerference than BPSK modulation. Thus, in alow signal-to-interference ratio (SIR) situation,the (3, 1) system with BPSK modulation is thebest implementation option, whereas in a highsignal-to-interference ratio (for example SIR =48 dB) situation, the (1, 1) system with 8-PSKmodulation is the best choice.

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[2] Furthermore, the SM decoder used in this paper is the maximum likelihood decoder (MLD) [2].

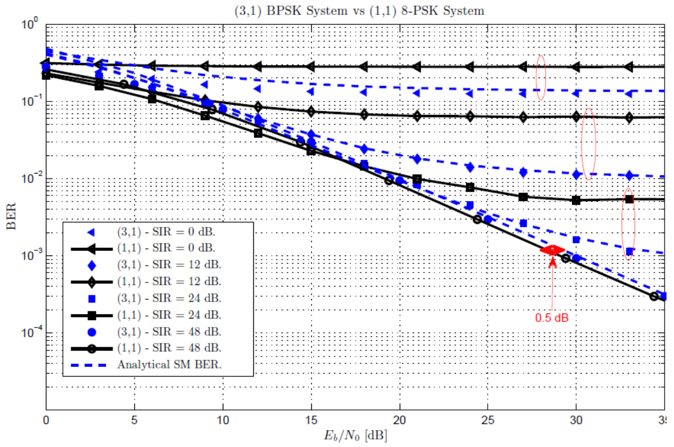

Figure 3. BER as a function of E_b/N_0 and SIR, for(3, 1) system with BPSK and (1, 1) system with 8-PSK modulation in the presence of one Co-Channel Interferer.

Fig.4 shows the BER as a function of $E_b/N_0$and SIR for a (4, 1) system with BPSK modulationversus a (1, 1) system with 16-QAMmodulation. Both systems have the same transmitpower and spectral efficiency equal to $4\,bits/s/H_z$. The curves and conclusions aresimilar to the previous case. We would like toemphasizing that for a SIR = 48, the (1, 1) hasan E_b/N_0advantage about 2 dB in comparisonto the (4, 1) system.

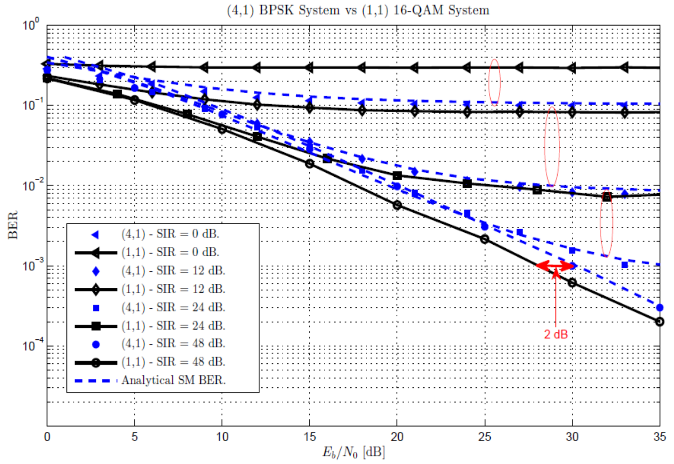

Figure 4. BER as a function of E_b/N_0 and SIR, for(4, 1) system with BPSK and (1, 1) system with 16-QAM modulation in the presence of one Co-Channel Interferer.

Fig.5 shows the performance comparison interms of the BER between a (2, 1) system withQPSK modulation and a (1, 1) system with 16-QAM modulation. The both systems have thesame transmit power and spectral efficiencyequal to $4\,bits/s/H_z$. In both systems, one dominantco-channel interferer is considered withSIR = 0, 12, 24, 48 dB. For a SIR = 48 dB,the BER in both systems is mainly limited bythe Rayleigh fading effects.

Thus, we can seethat the (2, 1) system has a better performance,by presenting an E_b/N_0 advantage of about1 dB in comparison with the (1, 1) system.Moreover, for low SIR values, it is observed significant signal degradation in both systemsdue to the effects of the co-cell interference.Thus, the BER in both systems presents floorsthat are independents of E_b/N_0increase.

Again,we notice that the (2, 1) system has a betterperformance in terms of BER that the (1, 1)system for all SIR values considered. Therefore, the (2, 1) system with QPSK modulation is the best implementation option over (1, 1) systemwith 16-QAM modulation.

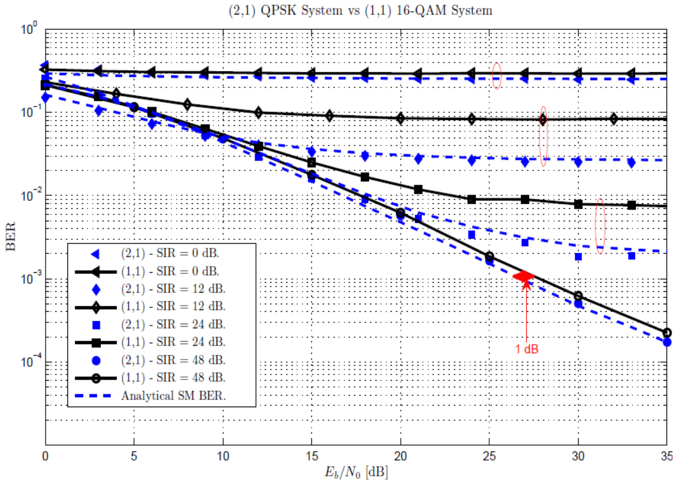

Figure 5. BER as a function of E_b/N_0 and SIR, for(2, 1) system with QPSK and (1, 1) system with 16-QAM modulation in the presence of one Co-Channel Interferer.

Fig.6 shows the BER as a function of $E_b/N_0$and SIR for a (3, 1) system with QPSK modulationversus (1, 1) system with 64-QAM modulation.The systems considered are the sametransmission power and same spectral efficiencyequal to $6\,bits/s/H_z$. We can see that thecurves and conclusions are similar to the abovecase (Figure 5). Note that for a SIR = 48 dB, the(3, 1) system with QPSK has an E_b/N_0 of about4 dB over the (1, 1) system with 64-QAM.

Thus, we would like to emphasize that aSM system with QPSK modulation, regardlessof the number of transmission antennas used,always performs better in terms of the BER thana non-SM with M-QAM modulation becomingthe best choice of implementation.

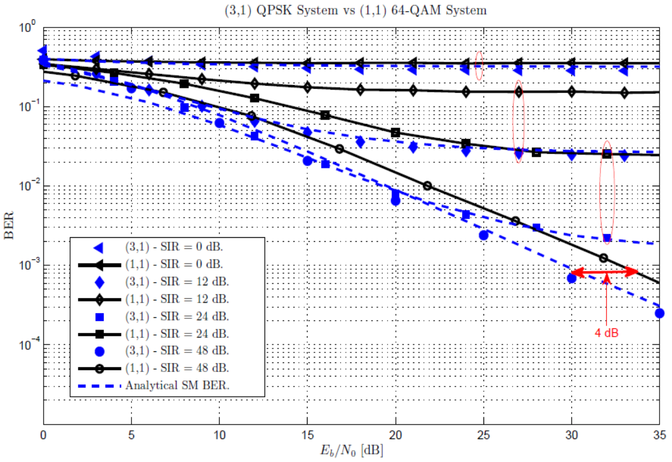

Figure 6. BER as a function of E_b/N_0 and SIR, for(3, 1) system with QPSK and (1, 1) system with 64-QAM modulation in the presence of one Co-Channel Interferer.

V. CONCLUSIONS

In this work, the SM's BER results were comparedwith a non-SM with same transmitpower and spectral efficiency, for which, a comparisonmethodology was proposed, in order tomake a fair comparison between both systems.In both systems the fading and CCI were considered.

We have shown that the presence of CCIcauses significant degradation in the performanceof SM and non-SM systems, presentingfloor in the BER curves. Further, from the obtainedresults we can conclude the following:

- A SM system with 3 and 4 transmissionantennas and BPSK modulation forlow SIR values presents less degradationwhen compared to non-SM system with8-PSK and 16-QAM modulation, respectively,due that the BSPK modulation ismore robust to the CCI effects, whereasfor high SIR values, the non-SM systemhave an E_b/N_0 advantage over the SMsystem with 3 and 4 transmission antennasand BPSK modulation. On the otherhand, for all SIR values considered inthis paper, a non-SM system with QPSKmodulation always presents a better performancethan SM system with 2 transmissionantennas and BPSK modulation.
- The SM systems with QPSK modulationare less susceptible to the CCI effects incomparison with a non-SM system withhigh order modulation *M*-QAM. In thisway, the SM systems with QPSK modulationalways present better performancein terms of BER for all SIR values considered.

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