On Extreme Point Complementary Quadratic Fractional Programming Problem

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Abstract—**In this paper, deals with the problem of optimizing the ratio of two quadratic functions subject to a set of linear constraints and the complementary condition with the additional restriction that the optimal solution should also be an extreme point of a second convex polyhedron. i.e., Extreme point complementary quadratic fractional Programming Problem (EPCQFPP) and consequently a convergent algorithm has been developed in the following discussion. Numerical examples have been provided in support of the theory. By using Matlab 2011 version 7.12.0.635 (R2011a).**

Keywords- EPCQFPP, Solving EPCQFPP by new technique to modified simplex method, New CP.

I. Introduction

Recently, in (1969-1971), Ibaraki studied complementary programming, and defined a new type of optimization problems, known as complementary programming problems ([26], [27]). A lot of work has been done in an extreme point linear programming problem which were investigated by Swarup, Kirby and Love in (1972) a special type of problems was studied, developed and solved that special type of programming problem in which the objective function is to be optimized over convex polyhedron with an additional condition that optimal solution is an extreme point of another convex polyhedron and a cutting plane algorithm for extreme point mathematical programming ([10], [11]) and Gupta, Swarup in (1973, 1978) ([16], [17]). In (2008) Hamad-Amin [2]. Also an extreme point linear fractional functional programming problem was attracted Puri and Swarup (1973, 1974) ([12], [13]). In (1985), Sen and Sherali [5] studied a branch and bound algorithm for an extreme point for the mathematical programming problem. Khurana and Arora (2011), are studied a quadratic fractional programming with linear homogenous constraints [3]. In (1988), Al-Barzinji [21] studied an extreme point optimization technique in mathematical programming. In (1989), Sulaiman [14] studied the computational aspects of single-objective indefinite quadratic programming problem with extreme point. In which the product of two linear functions is optimized subject to best a set of linear inequalities with the additional restriction that the optimum solution should be an extreme point of another convex polyhedron formed by another set of linear constraints also studied to quadratic complementary programming algorithms and their computer applications [14]. Gupta and Puri (1994) [18] studied extreme point quadratic fractional programming problem. In (2001) Fang, Lin and Wu [20] studied the cutting plane approach for solving quadratic semi-infinite problems. In (2009) Caraus and Necoara, were studied the cutting plane method for solving convex optimization problems [6]. In (1982) Gupta and Sharma had developed an algorithm for solving a quadratic complementary programming problem with indefinite [1]. In (2009) Arora and Narang are studies a bilevel fractional programming problem with independent followers [19]. In (2011) Judice studies algorithms for linear programming with linear complementarity constraints [7]. In (2010) Jahan and Islam [23] are studied a complementary slackness theorem for linear fractional programming problem.

In (1978) Garg and Swarup [9] are studies linear fractional functional complementary programming with extreme point optimization. In (2009) Fang, Gao, Sheu and Xing are studies to global optimization for class of fractional programming problems and quadratic fractional programming problem [22]. In (2011) Arora and Arora are studies to solving linear quadratic bilevel programming problem using Kuhn-Tucker conditions [15]. To extend this work, we have been defined EPCQFPP and investigated new cutting plane (CP) technique to generate the best compromising optimal solution.

II. Extreme Point CQFPP

Garg and Swarup [9] defined an EPCLFPP. The EPCQFPP can be formulated and defined as follows:

$$\left.\begin{array}{l} Max.Z = \dfrac{(c^T x + S^T u + s^T v + \gamma)(e^T x + O^T u + o^T v + \delta)}{(d^T x + W^T u + w^T v + \beta)(f^T x + G^T u + g^T v + \varepsilon)} \\ \text{Subject to:} \\ \qquad Ax + Bu + Cv = h \\ \qquad uv = 0 \\ \text{And } x, u, v \text{ is an extreme point of:} \\ \qquad Dx + Eu + Fv = k \\ \qquad x, u, v \geq 0 \end{array}\right\} \quad (1)$$

Where x, u, v are n, m, m-dimensional vectors of variables; c, e, f, d are n-dimensional and S, O, W, G, s, o, w, g are m-

dimensional vectors of constants; h and k are m, p-dimensional vectors of constants; A, B, C and D, E, F are matrices of order $m \times n, m \times m, m \times m$ and $p \times n$, $p \times m, p \times m$ matrix of constraints respectively, $\gamma, \delta, \beta, \varepsilon$ are constants. In order to solve the problem (1), related CQFPP are considered and we have to solve the following problem

$$Max. Z = \frac{(c^T x + S^T u + s^T v + \gamma)(e^T x + O^T u + o^T v + \delta)}{(d^T x + W^T u + w^T v + \beta)(f^T x + G^T u + g^T v + \varepsilon)}$$

Subject to:

$$\left.\begin{aligned} &Lx + Mu + Nv = I \\ &uv = 0 \\ &x, u, v \geq 0 \end{aligned}\right\} \quad (2)$$

Where $L = \binom{A}{D}$ is an order $(m+p) \times n$; $M = \binom{B}{E}$ is an order $(m+p) \times m$; $N = \binom{C}{F}$ is an order $(m+p) \times m$ matrix respectively and $I = \binom{h}{k}$ is an order $(m+p)$-dimensional vector. Since any solution is an extreme point of $Dx + Eu + Fv = k$; $x, u, v \geq 0$ and all the extreme points are finite, problem (1) is bounded. However problem (2) may or may not be bounded. We shall assume without any loss of generality that problem (2) is bounded, since addition of a single constraint linear part of $Z \leq K$, where K is an arbitrary large positive number, will provide us with a bounded problem and K being sufficiently large positive number, none of the extreme points of problem (1) are excluded from problem (2). Notations used let

J = Set of all non-zero columns of (D, E, F). $t = (x, u, v)$
$J(t)$ = Set of all the columns of J associated with non-zero variables only.
$S_1 = \{t: Ax + Bu + Cv = h$ and t is an extreme point of $Dx + Eu + Fv = k;\ x, u, v \geq 0\}$
$S_2 = \{t: (x, u, v)$ is an extreme point of $Lx + Mu + Nv = I;\ x, u, v \geq 0\}$. $S_3 = \{t: Lx + Mu + Nv = I; x, u, v \geq 0\}$
$T_1 = Y_1 = \{t_{11}, t_{12}, \ldots, t_{1k_1}\}$ is the set of all the optimal extreme point solutions of problem (2). V_1 = Value of the objective function corresponding to the optimal solution Y_1 of the problem (2). We know Puri and Swarup in (1974) [13], Swarup, Kirby and Love in (1972) [10], Garg and Swarup in (1978) [9] that $S_1 \subseteq S_2$, i.e., every extreme point of $Dx + Eu + Fv = k;\ x, u, v \geq 0$ which is feasible for $Ax + Bu + Cv = h$ is also an extreme point of $Lx + Mu + Nv = I;\ x, u, v \geq 0$. Thus our procedure for solving problem (1) involves working with problem (2).

III. Modified Simplex Method Development

Simplex method is developed by Dantzig in (1947). The simplex method provides a systematic algorithm which consists of moving from one basic feasible solution (one vertex) to another in prescribed manner such that the value of the objective function is improved. This procedure of jumping from vertex to vertex is repeated. If the objective function is improved at each jump, then no basis can ever be repeated and there is no need to go back to vertex already covered. Since the number of vertices is finite, the process must lead to the optimal vertex in a finite number of steps. The simplex algorithm is an iterative (step by step) procedure for solving linear programming problems. It consists of:

a) Having a trail basic feasible solution to constraint equations.
b) Testing whether is an optimal solution.
c) Improving the first trial solution by a set of rules, and repeating the process till an optimal solution is obtained.

For more details [24]. Modified simplex method to solve linear fractional programming problem and to solve quadratic objective function can be written as the produced two linear functions (QPP) [25]. Using new technique to modified simplex method to solve the numerical example to apply simplex process [25]. First we find $\Delta_{j1}, \Delta_{j2}, \Delta_{j3}$ and Δ_{j4} from the coefficients of numerator and denominator of objective function respectively, by using the following formula:

$\Delta_{ji} = C_{ji} - C_{Bi} x_{ji},\ i = 1,2,3,4,\ j = 1,2, \ldots, m+n,$
$z_1 = C_{B1} V_B + \gamma,\ z_2 = C_{B2} V_B + \delta,$
$z_3 = C_{B3} V_B + \beta,\ z_4 = C_{B4} V_B + \varepsilon,$
$\gamma, \delta, \beta, \varepsilon$ are constants, $Z_1 = z_1 z_2, Z_2 = z_3 z_4,\ \ Z = \frac{Z_1}{Z_2}$

$\mu_{j1} = \mu_{j2} = \min\left[\frac{V_B}{x_j}, x_j > 0\right]$ for non − basic variables
$\Delta_{ja} = z_1 \Delta_{j2} + z_2 \Delta_{j1} + \mu_{j1} \Delta_{j1} \Delta_{j2},$
$\Delta_{jb} = z_3 \Delta_{j4} + z_4 \Delta_{j3} + \mu_{j2} \Delta_{j3} \Delta_{j4}$

In this approach we define the formula to find Δ_j from Z_1, Z_2, Δ_{ja} and Δ_{jb} as follows: $\Delta_j = Z_2 \Delta_{ja} - Z_1 \Delta_{jb}$. Here C_{ji} are the coefficients of the basic and non-basic variables in the objective function and, C_{Bi} are the coefficients of the basic variables in the objective function, $j = 1,2, \ldots, m+n$, $i = 1,2,3,4$. For testing optimality solution must be all $\Delta_j \leq 0$ but here all Δ_j not lesser than zero, and then the solution is not optimal. Repeat the same approach to find next feasible solution.

IV. Algorithm For EPCQFPP

The following algorithm is to obtain the optimal solution for the EPCQFPP by new technique to modified simplex method which can be summarized as follows:

Step1: To start with, solve problem (2) without the complementary condition which is an ordinary QFPP by new technique to modified simplex method ([4], [24], [25]). Examine whether the complementary condition, $uv = 0$, is satisfied.

Step2: If $uv = 0$, go to step4. If not, go to step3.

Step3: If the complementary condition $uv = 0$ is not satisfied, use the branch and bound method proposed by the authors Garg and Swarup in (1977) [8] and ensure that $uv = 0$. Let T_1 be the set of all optimal extreme point solutions of problem (2). Let V_1 be the value of the objective function corresponding to the solution T_1. Go to step4.

Step4: If $uv = 0$, examine whether T_1, the set of all optimal extreme point solutions of problem (2), is also an extreme point of problem (1) i.e. $T_1 \cap S_1 \neq \emptyset$. For this we shall proceed as follows. Let the number of columns in $J(t)$ be p^*. If $p^* = p$, we declare that T_1, the set of optimal extreme point solutions of problem (2) is an extreme point solution of problem (1) i.e. $T_1 \cap S_1 \neq \emptyset$. Terminate here. If $p^* \neq p$, $T_1 \cap S_1 = \emptyset$. Go to step5.

Step5: If $T_1 \cap S_1 = \emptyset$, consider the following problem (3)

$$\left.\begin{array}{l} Max. Z = \frac{(c^T x + s^T u + s^T v + \gamma)(e^T x + o^T u + o^T v + \delta)}{(d^T x + W^T u + w^T v + \beta)(f^T x + G^T u + g^T v + \varepsilon)} \\ \text{Subject to:} \\ \quad Lx + Mu + Nv = I \\ \quad z_1 \leq v_{11} \text{ if } v_{11} \geq v_{12},\ v_{11} \geq v_{13},\ v_{11} \geq v_{14},\ v_{11} \geq V_1 \\ \text{Or } z_2 \leq v_{12} \text{ if } v_{12} \geq v_{11},\ v_{12} \geq v_{13},\ v_{12} \geq v_{14},\ v_{12} \geq V_1 \\ \text{Or } z_3 \leq v_{13} \text{ if } v_{13} \geq v_{11},\ v_{13} \geq v_{12},\ v_{13} \geq v_{14},\ v_{13} \geq V_1 \\ \text{Or } z_4 \leq v_{14} \text{ if } v_{14} \geq v_{11},\ v_{14} \geq v_{12},\ v_{14} \geq v_{13},\ v_{14} \geq V_1 \\ \quad uv = 0 \\ \quad x, u, v \geq 0 \end{array}\right\} \quad (3)$$

Solve problem (3) without the complementary condition. Repeat step1 to step4 again. Since the extreme points of $Dx + Eu + Fv = k; x, u, v \geq 0$ are finite, therefore, the procedure will come to an end after a finite number of steps. Suppose the procedure terminates at the i-th stage. Let T_i, the i-th best extreme point solution of problem (2), be also an extreme point solution of problem (1) i.e. $T_1 \cap S_1 \neq \emptyset$. Let V_1 be the value of the objective function corresponding to the i-th best extreme point solution of problem (2).Terminate.

V. NUMERICAL EXAMPLES AND RESULTS

In this section we have presented only two examples among several examples that we have solved, to show the accuracy of the new technique and which new technique to modified simplex method is better and more convenience.

EXAMPLE I**:** We consider the following EPCQFPP as

$$Max. Z = \frac{(5x_1 + 5x_2)(3x_1 + 3x_2)}{(x_1 + x_2 + 1)(2x_1 + 2x_2 + 2)}$$

Subject to:

$$2x_1 + x_2 \leq 8$$
$$x_1 x_2 = 0$$

And x_1, x_2 is an extreme point of:

$$3x_1 + x_2 \leq 9$$
$$x_1 + x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

SOLUTION I: Solving the example I by new technique to modified simplex method and applied an algorithm in the section IV, after 2 steps we obtained the initial table as follows in table I. After three iterations, we obtained the result in the following table II

TABLE I. INITIAL TABLE FOR EXAMPLE I BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}						
					C_{j2}						
					C_{j3}						
					C_{j4}						
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	Min ratio
x_3	0	0	0	0	8	2	1	1	0	0	8/1 = 8
x_4	0	0	0	0	9	3	1	0	1	0	9/1 = 9
x_5	0	0	0	0	8	1	1	0	0	1	8/1 = 8
$z_1 = 0$					Δ_{j1}	5	5	0	0	0	
$z_2 = 0$					Δ_{j2}	3	3	0	0	0	
$Z_1 = z_1 z_2 = 0$					μ_{j1}	3	8	0	0	0	
$z_3 = 1$					Δ_{j3}	1	1	0	0	0	
$z_4 = 2$					Δ_{j4}	2	2	0	0	0	
$Z_2 = z_3 z_4 = 2$					μ_{j2}	3	8	0	0	0	
$Z_1 = 0$					Δ_{ja}	45	120	0	0	0	
$Z_2 = 2$					Δ_{jb}	10	20	0	0	0	
$Z = \frac{Z_1}{Z_2} = 0$					Δ_j	90	240	0	0	0	

TABLE II. FINAL TABLE FOR EXAMPLE I BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}	5	5	0	0	0	
					C_{j2}	3	3	0	0	0	
					C_{j3}	1	1	0	0	0	
					C_{j4}	2	2	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	Min ratio
x_2	5	3	1	2	8	1	1	0	0	1	
x_3	0	0	0	0	0	1	0	1	0	−1	
x_4	0	0	0	0	1	2	0	0	1	−1	
$z_1 = 40$					Δ_{j1}	0	0	0	0	−5	
$z_2 = 24$					Δ_{j2}	0	0	0	0	−3	
$Z_1 = z_1 z_2 = 960$					μ_{j1}	0	0	0	0	8	
$z_3 = 9$					Δ_{j3}	0	0	0	0	−1	
$z_4 = 18$					Δ_{j4}	0	0	0	0	−2	
$Z_2 = z_3 z_4 = 162$					μ_{j2}	0	0	0	0	8	
$Z_1 = 960$					Δ_{ja}	0	0	0	0	−120	
$Z_2 = 162$					Δ_{jb}	0	0	0	0	−20	
$Z = \frac{Z_1}{Z_2} = \frac{960}{162}$					Δ_j	0	0	0	0	−240	

After solving it by new technique to modified simplex method, without complementary condition $(x_1 x_2 = 0)$, we get $Max. Z = \frac{960}{162}$ and $x_1 = 0,\ x_2 = 8$. Since $x_1 x_2 = 0$

This solution is an extreme point of:

$$3x_1 + x_2 \leq 9$$
$$x_1 + x_2 \leq 8$$

And feasibility to: $2x_1 + x_2 \leq 8$

EXAMPLE II: We consider the following EPCQFPP as

$$Max.Z = \frac{(x_1+2x_2)(2x_1+x_2)}{(x_1+x_2+3)(2x_1+2x_2+4)}$$

Subject to:

$$3x_1 + x_2 \leq 9$$
$$x_1 x_2 = 0$$

And x_1, x_2 is an extreme point of:

$$2x_1 + x_2 \leq 8$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

SOLUTION II: Solving the example II by new technique to modified simplex method and applied an algorithm in the section IV, after 2 steps we obtained the initial table as follows in table I. After two iterations, we obtained the result in the following table II

TABLE I. INITIAL TABLE FOR EXAMPLE II BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}	1	2	0	0	0	
					C_{j2}	2	1	0	0	0	
					C_{j3}	1	1	0	0	0	
					C_{j4}	2	2	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	Min ratio
x_3	0	0	0	0	9	3	1	1	0	0	$9/1 = 9$
x_4	0	0	0	0	8	2	1	0	1	0	$8/1 = 8$
x_5	0	0	0	0	12	2	3	0	0	1	$12/3 = 4$
$z_1 = 0$					Δ_{j1}	1	2	0	0	0	
$z_2 = 0$					Δ_{j2}	2	1	0	0	0	
$Z_1 = z_1 z_2 = 0$					μ_{j1}	3	4	0	0	0	
$z_3 = 3$					Δ_{j3}	1	1	0	0	0	
$z_4 = 4$					Δ_{j4}	2	2	0	0	0	
$Z_2 = z_3 z_4 = 12$					μ_{j2}	3	4	0	0	0	
$Z_1 = 0$					Δ_{ja}	6	8	0	0	0	
$Z_2 = 12$					Δ_{jb}	16	18	0	0	0	
$Z = \frac{Z_1}{Z_2} = 0$					Δ_j	72	96	0	0	0	

TABLE II. FINAL TABLE FOR EXAMPLE II BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}	1	2	0	0	0	
					C_{j2}	2	1	0	0	0	
					C_{j3}	1	1	0	0	0	
					C_{j4}	2	2	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	Min ratio
x_1	1	2	1	2	15/7	1	0	$3/7$	0	$-1/7$	
x_4	0	0	0	0	8/7	0	0	$-4/7$	1	$-1/7$	
x_2	2	1	1	2	18/7	0	1	$-2/7$	0	$3/7$	
$z_1 = 51/7$					Δ_{j1}	0	0	$1/7$	0	$-5/7$	
$z_2 = 48/7$					Δ_{j2}	0	0	$-4/7$	0	$-1/7$	
$Z_1 = z_1 z_2 = \frac{2448}{49}$					μ_{j1}	0	0	5	0	6	
$z_3 = 54/7$					Δ_{j3}	0	0	$-1/7$	0	$-2/7$	
$z_4 = 94/7$					Δ_{j4}	0	0	$-2/7$	0	$-4/7$	
$Z_2 = z_3 z_4 = \frac{5076}{49}$					μ_{j2}	0	0	5	0	6	
$Z_1 = \frac{2448}{49}$					Δ_{ja}	0	0	$-176/49$	0	$-261/49$	
$Z_2 = \frac{5076}{49}$					Δ_{jb}	0	0	$-192/49$	0	$-356/49$	
$Z = \frac{Z_1}{Z_2} = \frac{2448}{5076}$					Δ_j	0	0	$\frac{-423360}{2401}$	0	$\frac{-453348}{2401}$	

After solving it by new technique to modified simplex method, without complementary condition $(x_1 x_2 = 0)$, we get $Max.Z = \frac{2448}{5076}$ and $x_1 = \frac{15}{7}$, $x_2 = \frac{18}{7}$

Since the point $(\frac{15}{7}, \frac{18}{7})$, is not an extreme point to the set constraints:

$$2x_1 + x_2 \leq 8$$
$$2x_1 + 3x_2 \leq 12$$

So, using cutting plane technique by adding the linear constraint:

$$2x_1 + 2x_2 \leq \frac{66}{7}$$

The problem will become as follows:

$$Max.Z = \frac{(x_1+2x_2)(2x_1+x_2)}{(x_1+x_2+3)(2x_1+2x_2+4)}$$

Subject to:

$$3x_1 + x_2 \leq 9$$
$$x_1 x_2 = 0$$

And x_1, x_2 is an extreme point of:

$$2x_1 + x_2 \leq 8$$
$$2x_1 + 3x_2 \leq 12$$
$$2x_1 + 2x_2 \leq \frac{66}{7}$$
$$x_1, x_2 \geq 0$$

TABLE I. INITIAL TABLE FOR EXAMPLE II AFTER CUTTING PLANE TECHNIQUE SOLVING BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}	1	2	0	0	0	0	
					C_{j2}	2	1	0	0	0	0	
					C_{j3}	1	1	0	0	0	0	
					C_{j4}	2	2	0	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_3	0	0	0	0	9	3	1	1	0	0	0	$9/1 = 9$
x_4	0	0	0	0	8	2	1	0	1	0	0	$8/1 = 8$
x_5	0	0	0	0	12	2	3	0	0	1	0	$12/3 = 4$
x_6	0	0	0	0	66/7	2	2	0	0	0	1	$\frac{66/7}{2} = 66/14$
$z_1 = 0$					Δ_{j1}	1	2	0	0	0	0	
$z_2 = 0$					Δ_{j2}	2	1	0	0	0	0	
$Z_1 = z_1 z_2 = 0$					μ_{j1}	3	4	0	0	0	0	
$z_3 = 3$					Δ_{j3}	1	1	0	0	0	0	
$z_4 = 4$					Δ_{j4}	2	2	0	0	0	0	
$Z_2 = z_3 z_4 = 12$					μ_{j2}	3	4	0	0	0	0	
$Z_1 = 0$					Δ_{ja}	6	8	0	0	0	0	
$Z_2 = 12$					Δ_{jb}	16	18	0	0	0	0	
$Z = \frac{Z_1}{Z_2} = 0$					Δ_j	72	96	0	0	0	0	

TABLE II. FINAL TABLE FOR EXAMPLE II AFTER CUTTING PLANE TECHNIQUE SOLVING BY NEW TECHNIQUE TO MODIFIED SIMPLEX METHOD

					C_{j1}	1	2	0	0	0	0	
					C_{j2}	2	1	0	0	0	0	
					C_{j3}	1	1	0	0	0	0	
					C_{j4}	2	2	0	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_1	1	2	1	2	15/7	1	0	3/7	0	−1/7	0	
x_4	0	0	0	0	8/7	0	0	−4/7	1	−1/7	0	
x_2	2	1	1	2	18/7	0	1	−2/7	0	3/7	0	
x_6	0	0	0	0	0	0	0	−2/7	0	−4/7	1	
$z_1 = 51/7$					Δ_{j1}	0	0	1/7	0	−5/7	0	
$z_2 = 48/7$					Δ_{j2}	0	0	−4/7	0	−1/7	0	
$Z_1 = z_1 z_2 = \frac{2448}{49}$					μ_{j1}	0	0	5	0	6	0	
$z_3 = 54/7$					Δ_{j3}	0	0	−1/7	0	−2/7	0	
$z_4 = 94/7$					Δ_{j4}	0	0	−2/7	0	−4/7	0	
$Z_2 = z_3 z_4 = \frac{5076}{49}$					μ_{j2}	0	0	5	0	6	0	
$Z_1 = \frac{2448}{49}$					Δ_{ja}	0	0	−176/49	0	−261/49	0	
$Z_2 = \frac{5076}{49}$					Δ_{jb}	0	0	−192/49	0	−356/49	0	
$Z = \frac{Z_1}{Z_2} = \frac{2448}{5076}$					Δ_j	0	0	$\frac{-423360}{2401}$	0	$\frac{-453348}{2401}$	0	

After solving it by new technique to modified simplex method, without complementary condition $(x_1 x_2 = 0)$, we get $Max. Z = \frac{2448}{5076}$ and $x_1 = \frac{15}{7}$, $x_2 = \frac{18}{7}$
This solution is an extreme point of:

$$2x_1 + x_2 \leq 8$$
$$2x_1 + 3x_2 \leq 12$$
$$2x_1 + 2x_2 \leq \frac{66}{7}$$

And feasibility of: $3x_1 + x_2 \leq 9$

This is fractional in x_1 and x_2. So, the best integral solution which we can obtain by using the Branch-and-Bound Procedure method is $Max. Z = \frac{36}{84}$ and $x_1 = 2,\ x_2 = 2$

Now, we are going to satisfied the complementary condition $(x_1 x_2 = 0)$, because $x_1 x_2 \neq 0$, if $x_1 = 0$, then the optimal solution is $Max. Z = \frac{8}{40}$ and $x_1 = 0,\ x_2 = 2$

VI. DISCUSSION

In this paper find EPCQFPP by the new technique to modified simplex method. The optimal solution must be at one of the extreme points complementary of the polygon of the feasible region, sometimes it may be need to use new cutting plane technique for finding best extreme point complementary solution for the problem.

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