

# A Solution to Multi Criteria Robot Selection Problems Using Grey Relational Analysis

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**Abstract**— Grey relational analysis (GRA) is a vital instrument suitable for optimal selection which can be used for modelling, forecasting and decision making. Grey theory gives reliable solution of systems in which the model is poor with little or incomplete information. This paper aims to use GRA for solving Multi Criteria Robot Selection Problems (MCRSPs). In this paper, GRA steps are implemented using a fast computational tool on two practical cases and results were compared with previous methodologies to confirm the validity of the GRA approach. Results show that the distinguishing coefficient has minimal impact on the GRA solution, thereby making this approach appropriate for accurate modelling of MCRSPs.

**Keywords**- MCRSPs; grey theory; grey relational analysis; distinguishing coefficient

## I. INTRODUCTION

Over the years, rapid growth in computers and applied sciences has led to an explosion of scientific innovations amongst which robots, computer aided machines and other automated systems belong. As technology becomes more sophisticated, the role of man in the manufacturing process is gradually replaced with industrial robots. Robots can function in environments dangerous to humans such as radioactive, heated, toxic and noisy settings. Due to the high cost of acquiring and implementing these industrial robots, manufacturers often seek an optimal solution.

It is observed that there are so many mutually conflicting performance criteria, like dynamic accuracy, repeatability, speed, load capacity, program flexibility, handling coefficient, memory capacity, manipulator reach, supplier's service quality etc. that influence the robot selection decision [1]. Dynamic accuracy and repeatability is the ability of a robot to follow a desired trajectory with little or no variance. Speed is how fast a robot can position itself. Carrying capacity refers to how much weight a robot can lift. Memory capacity is the capacity to store the steps of a predefined program in memory by a robot. Manipulator reach is defined as the boundary in which the manipulator of a robot can reach. Considering these criteria listed, some are advantageous while others are not. The load capacity, memory capacity, flexibility in robot program and manipulator reach are advantageous criteria where higher values are desired, while repeatability and cost are non-advantageous in nature, that is, lower values are desired.

As market for robots is on the rise, it becomes a difficult task to make a selection decision on the appropriate robot to be deployed for optimal results. A rigorous performance check is essential, in which the effect of various selection criteria is examined. Several approaches including multi criteria decision making (MCDM) approaches and optimization techniques have previously been proposed by the earlier researchers for robot selection. The present paper proposes using grey relational analysis (GRA) for solving MCRSPs accurately and faster.

In [4], an analytic network process and mixed integer goal programming (MIGP) model was used to select robot for a computer integrated manufacturing system. The model considered multi criteria, interdependence property and optimization for selecting robots. A unique multiplicative model and algorithmic approach was proposed in [6] and [5], [7] respectively. The use of Choquet integral based decision making method was employed in [13], previously published data set was used to in the study and results were compared to previous approaches. The data envelopment analysis (DEA) approach used in [1] was computationally complex. Electre II method is a time consuming approach used to discard alternatives that are unacceptable and uses a different MCDM approach to make selection.

Grey relational analysis was presented in [8] using interval fuzzy numbers, where interval valued indices are used to apply multiplicative operations in place of interval numbers and [3], [11] aimed at developing a fuzzy MCDM model to solve complicated systems with multiple objectives. [9], [10] and [12] used grey relational analysis in solving selection problems using various applications.

It is observed that previous researchers used different approaches to solve the robot selection problem. The VIKOR method in [15] made ranking selection of conflicting criteria based on closeness to ideal solution. The GRA methods adopted in [9], [10] and [12] was effectively applied to solve variety of problems, but failed to consider MCRSPs. The GRA is mathematically comprehensible and less rigorous than other methodologies. Thus, this paper presents a faster and efficient solution to MCRSPs using the GRA and examines the effect of using various distinguishing coefficient on GRA results.

The paper is organized as follows. In Section II, the grey system theory and proposed scheme is discussed. Section III presents experimentation with two robot selection problems. Discussion and results are presented in section IV. Finally section V is the conclusion.

## II. GREY SYSTEM THEORY AND SCHEME

Grey system theory was introduced by Deng Ju-Long in 1982 [10]. Grey system theory is built on the notion that a system is uncertain, and that the information contained the system is inadequate to construct a reliable model that describes the system [9]. This can be clearly seen in Fig. 1. This suggests that grey models are appropriate for predicting future events where manufacturers can make use of very limited older data. This study will lay emphasis on the GRA.

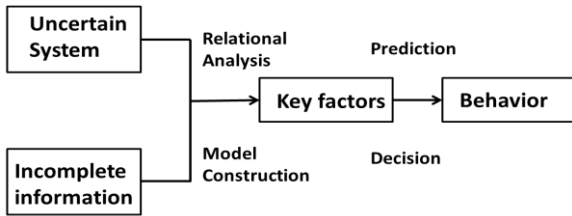


Figure 1. Grey system flow

The GRA can be clearly broken down into four steps, namely, grey relational generation, a reference sequence generation, grey relational coefficient calculation and grey relational grade calculation. These steps are further explained:

### A. Grey relational generating (GRG)

GRG is a normalization process where all performance attributes are processed into a comparable sequence. Equation (1) is used to normalize the higher the better attributes, (2) for lower the better and for the closer to the desired the better attributes, (3) is used for normalization. For MCDM problems,  $m$  is given as the robot alternatives and  $n$  as performance attributes. Given a robot selection problem  $Y_i = (y_{i1}, y_{i2}, \dots, y_{ij}, \dots, y_{in})$ , we can deduce the comparability sequence  $X_i = (x_{i1}, x_{i2}, \dots, x_{ij}, \dots, x_{in})$ . For all  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

$$x_{ij} = \frac{y_{ij} - \text{Min}\{y_{ij}, i=1,2,\dots,m\}}{\text{Max}\{y_{ij}, i=1,2,\dots,m\} - \text{Min}\{y_{ij}, i=1,2,\dots,m\}} \quad (1)$$

$$x_{ij} = \frac{\text{Max}\{y_{ij}, i=1,2,\dots,m\} - y_{ij}}{\text{Max}\{y_{ij}, i=1,2,\dots,m\} - \text{Min}\{y_{ij}, i=1,2,\dots,m\}} \quad (2)$$

$$x_{ij} = 1 - \frac{|y_{ij} - y_j^*|}{\text{Max}\{\text{Max}\{y_{ij}, i=1,2,\dots,m\} - y_j^*, y_j^* - \text{Min}\{y_{ij}, i=1,2,\dots,m\}\}} \quad (3)$$

### B. Reference sequence generation (RSG)

After the normalization process using GRG, all performance values are defined within the range [0, 1]. If the value  $x_{ij}$  with an attribute  $j$  of alternative  $i$ , which equals 1 or approaches 1, it implies that the performance of alternative  $i$  is the most suitable for attribute  $j$ . The reference sequence is given as  $X_0 = (x_{01}, x_{02}, \dots, x_{0j}, \dots, x_{0n})$  and the study sets the sequence as all ones (1, 1, ..., 1, ..., 1) with the aim of finding the alternative whose  $X_i$  is closest to  $X_0$ .

### C. Grey relational coefficient calculation (GRC)

To determine how close the comparability sequence is to the reference sequence, we calculate the GRC. Equation (4) gives an expression of the grey relational coefficient.

The role of  $\zeta$  in (4) expands and compresses the range of GRC. This can clearly be seen in Fig. 2 and 3. The used  $\zeta=0.5$  for implementing the GRA steps and further tested the results

$$\gamma(x_{0j}, x_{ij}) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_{ij} + \zeta \Delta_{\max}} \quad (4)$$

Where  $\gamma(x_{0j}, x_{ij})$  is the GRC between  $x_{0j}$  and  $x_{ij}$ ,

$$\Delta_{ij} = |x_{0j} - x_{ij}|,$$

$$\Delta_{\min} = \text{Min}\{\Delta_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n\},$$

$$\Delta_{\max} = \text{Max}\{\Delta_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n\},$$

$\zeta$  = distinguishing coefficient.

with other distinguishing coefficient values.

### D. Grey relational grade calculation (GRGC)

After the grey relational coefficient is calculated, the grey relational grade between the  $X_i$  and  $X_0$  can then be calculated using (5).

$$\Gamma(X_0, X_i) = \sum_{j=1}^n w_j \gamma(x_{0j}, x_{ij}) \quad (5)$$

Where  $\Gamma(x_{0j}, x_{ij})$  is the GRG between  $x_{0j}$  and  $x_{ij}$ , the weight of attribute  $j$  is expressed as  $w_j$  and it is subject to the decision makers' judgment of a particular problem. An alternative that has the closest value to the reference value is ranked best.

## III. EXPERIMENT

In this section, two case studies were considered. The proposed GRA procedure is applied to provide a basis for a faster and accurate robot selection decision making process. Computational simulations were carried out using Matlab software.

### A. Case Study 1

The first case study deals with selection of industrial robots in a manufacturing environment. The data used in Table I was adapted from [1] where a DEA method was used in solving the

problem, considering repeatability (RE), load capacity (LC), maximum tip speed (MTS), memory capacity (MC) and manipulator reach (MR) as the predominant robot selection attributes. Out of the five performance attributes, only repeatability is non-advantageous and was normalized using (2). Other attributes are advantageous and were normalized using (1).

TABLE I. ROBOT SELECTION DECISION MATRIX FOR CASE STUDY 1 [1]

Robot	No.	LC (kg)	RE (mm)	MTS (mm/sec)	MC (steps)	MR (mm)
ASEA-IRB 60/2	1	60.0	0.40	2540.0	500	990
Cincinnati Milacrone	2	6.35	0.15	1016.0	3000	1041
Cybotech	3	6.8	0.10	1727.2	1500	1676
Hitachi America Process Robot	4	10.0	0.20	1000.0	2000	965
Unimation PUMA 500/600	5	2.5	0.10	560.0	500	915
US Robots Maker 110	6	4.5	0.08	1016.0	350	508
Yaskawa Electric Motoman L3C	7	3.0	0.10	1778.0	1000	920

In Table II, the study set the value of  $X_0 = \{1, 1, 1, 1, 1\}$  for the five performance attributes.

TABLE II. NORMALIZED DECISION MATRIX FOR CASE STUDY 1

Robot	No.	LC (kg)	RE (mm)	MTS (mm/s)	MC (steps)	MR (mm)
Reference Sequence, $X_0$	-	1.0000	1.0000	1.0000	1.0000	1.0000
ASEA-IRB 60/2	1	1.0000	0	1.0000	0.0566	0.3356
Cincinnati Milacrone	2	0.0670	0.7813	0.3551	1.0000	0.4563
Cybotech	3	0.0748	0.9375	0.6560	0.4340	1.0000
Hitachi America Process Robot	4	0.1304	0.6250	0.3483	0.6226	0.3913
Unimation PUMA 500/600	5	0	0.9375	0.1621	0.0566	0.3485
US Robots Maker 110	6	0.0348	1.0000	0.3551	0	0

Yaskawa Electric Motoman L3C	7	0.0087	0.9375	0	0.2453	0.3527
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For example, to calculate the values of  $\Delta_{ij}$ ,  $\Delta_{max}$ ,  $\Delta_{min}$ , we have,  $\Delta_{11} = |1-1| = 0$ ,  $\Delta_{max} = 1$  and  $\Delta_{min} = 0$ . Now,  $\gamma(x_{01}, x_{11}) = (0 + 0.5 \times 1)/(0 + 0.5 \times 1) = 1$

The study used a distinguishing coefficient of 0.5 to calculate the grey relational coefficient. Table III shows the results obtained.

TABLE III. GREY RELATIONAL COEFFICIENT MATRIX FOR CASE STUDY 1

Robot	No.	LC (kg)	RE (mm)	MTS (mm/sec)	MC (steps)	MR (mm)
ASEA-IRB 60/2	1	1.0000	0.3333	1.0000	0.3464	0.4294
Cincinnati Milacrone	2	0.3489	0.6957	0.4367	1.0000	0.4791
Cybotech	3	0.3508	0.8889	0.5924	0.4690	1.0000
Hitachi America Process Robot	4	0.3651	0.5714	0.4341	0.5699	0.4510
Unimation PUMA 500/600	5	0.3333	0.8889	0.3737	0.3464	0.4342
US Robots Maker 110	6	0.3412	1.0000	0.4367	0.3333	0.3333
Yaskawa Electric Motoman L3C	7	0.3353	0.8889	0.3333	0.3985	0.4358

Calculation for the grey relational analysis was done using (4) and the result is shown in column 3 of Table IV.

TABLE IV. RESULTS OF GREY RELATIONAL ANALYSIS FOR CASE STUDY 1

Robot	No.	Grey relational grade	Rank results of GRA	Vikor method [15]	Electre II method [2]
ASEA-IRB 60/2	1	0.6218	2	4	2
Cincinnati Milacrone	2	0.5921	3	2	3
Cybotech	3	0.6602	1	1	1
Hitachi America Process Robot	4	0.4783	6	3	6
Unimation PUMA 500/600	5	0.4753	7	5	7
US Robots Maker 110	6	0.4889	4	6	5
Yaskawa Electric Motoman L3C	7	0.4784	5	7	4

B. Case Study 2

Elaborating on the effectiveness and speed of GRA approach, a second case study is considered as shown in Table V. The study adapted a robot selection problem from [2] and [14]. Five parameter attribute were used on twelve robot alternative. Cost and repeatability were considered not advantageous, while handling capacity, load capacity and velocity were considered to be advantageous. The GRA steps were implemented on the data with results shown in Table VI, VII, VIII.

TABLE V. ROBOT SELECTION DECISION MATRIX FOR CASE STUDY 2 [14]

Robot	Cost (US\$)	HC	LC (kg)	Repeatability (mm)	Velocity (m/s)
1	100000	0.995	85.0	0.588	3.00
2	75000	0.933	45.0	0.400	3.60
3	56250	0.875	18.0	0.200	2.20
4	28125	0.409	16.0	0.588	1.50
5	46875	0.818	20.0	0.200	1.10
6	78125	0.664	60.0	0.400	1.35
7	87500	0.880	90.0	0.500	1.40
8	56250	0.633	10.0	0.125	2.50
9	56250	0.653	25.0	0.250	2.50
10	87500	0.747	100.0	0.500	2.50
11	68750	0.880	100.0	0.250	1.50
12	43750	0.633	70.0	0.200	3.00

In Table VI, the study set the value of  $X_0 = \{1, 1, 1, 1, 1\}$  for the five performance attributes.

TABLE VI. NORMALIZED DECISION MATRIX FOR CASE STUDY 2

Robot	Cost (US\$)	HC	LC (kg)	Repeatability (mm)	Velocity (m/s)
$X_0$	1.0000	1.0000	1.0000	1.0000	1.0000
1	0	1.0000	0.8333	0	0.7600
2	0.3478	0.8942	0.3889	0.4060	1.0000
3	0.6087	0.7952	0.0889	0.8380	0.4400
4	1.0000	0	0.0667	0	0.1600
5	0.7391	0.6980	0.1111	0.8380	0
6	0.3043	0.4352	0.5556	0.4060	0.1000
7	0.1739	0.8038	0.8889	0.1901	0.1200
8	0.6087	0.3823	0	1.0000	0.5600
9	0.6087	0.4164	0.1667	0.7300	0.5600
10	0.1739	0.5768	1.0000	0.1901	0.5600
11	0.4348	0.8038	1.0000	0.7300	0.1600
12	0.7826	0.3823	0.6667	0.8380	0.7600

For example, to calculate the values of  $\Delta_{ij}$ ,  $\Delta_{max}$ ,  $\Delta_{min}$ , we have,  $\Delta_{11} = |1-0| = 1$ ,  $\Delta_{max} = 1$  and  $\Delta_{min} = 0$ . Now,  $\gamma(x_{0j}, x_{1j}) = (0 + 0.5 \times 1)/(1 + 0.5 \times 1) = 0.3333$ .

The study used a distinguishing coefficient of 0.5 to calculate the grey relational coefficient. Table VII shows the results obtained.

TABLE VII. GREY RELATIONAL COEFFICIENT MATRIX FOR CASE STUDY 2

Robot	Cost (US\$)	HC	LC (kg)	Repeatability (mm)	Velocity (m/s)
1	0.3333	1.0000	0.7500	0.3333	0.6757
2	0.4340	0.8254	0.4500	0.4571	1.0000
3	0.5610	0.7094	0.3543	0.7553	0.4717

4	1.0000	0.3333	0.3488	0.3333	0.3731
5	0.6571	0.6234	0.3600	0.7553	0.3333
6	0.4182	0.4696	0.5294	0.4571	0.3571
7	0.3770	0.7181	0.8182	0.3817	0.3623
8	0.5610	0.4473	0.3333	1.0000	0.5319
9	0.5610	0.4614	0.3750	0.6494	0.5319
10	0.3770	0.5416	1.0000	0.3817	0.5319
11	0.4694	0.7181	1.0000	0.6494	0.3731
12	0.6970	0.4473	0.6000	0.7553	0.6757

Calculation for the grey relational analysis was done using (4) and the result is shown in the third column of Table VIII.

TABLE VIII. RESULTS OF GREY RELATIONAL ANALYSIS FOR CASE STUDY 2

Robot	Grey relational grade	Rank results of GRA	DEA [14]	Minimax efficiency [14]
1	0.6185	4	11	9
2	0.6333	3	7	6
3	0.5703	6	4	4
4	0.4777	11	5	5
5	0.5458	8	1	1
6	0.4463	12	12	12
7	0.5315	9	10	8
8	0.5747	5	1	10
9	0.5157	10	8	7
10	0.5665	7	9	11
11	0.6420	1	6	3
12	0.6351	2	1	1

IV. RESULTS AND DISCUSSION

Empirical results show that GRA offers reliable solutions when compared with the results obtained from existing methods. It has the capability of providing differences between alternatives. Most MCDM methods fail to provide this distinction, thus, may present multiple choices. This study examined the impact of the distinguishing coefficient on the grey relational grade as suggested in [12]. In case study 1, the behaviour of the system was alike when  $\zeta$  was set to values (0.1, 0.3, 0.5, 0.7, 0.9) as shown in Fig. 2, but at some instances slight changes in trends occurred. Alternative 3 was best with  $\zeta = 0.5$  and second best with  $\zeta = 0.1$ , indicating that the distinguishing coefficient has a minimal impact on the GRA results. This finding was further validated when case study 2's result was analysed using the same set of distinguishing coefficient values.

V. CONCLUSION

The two cases of robot selection examined show that the GRA methodology can derive quite acceptable and satisfactory ranking results to assist the decision makers in taking appropriate decisions. The results of this study reveal that the GRA approach is comprehensible and less rigorous in calculation since the approach is simple to implement. This approach gives a distinct ranking of alternatives when compared to other techniques. This plays a vital role in priority assignment. The uniform trend in various distinguishing coefficient used in the study indicates consistency in results

obtained. The methodology discussed can allow decision makers design a robot selection model for evaluation of alternatives in complex multi criteria decision making problems.

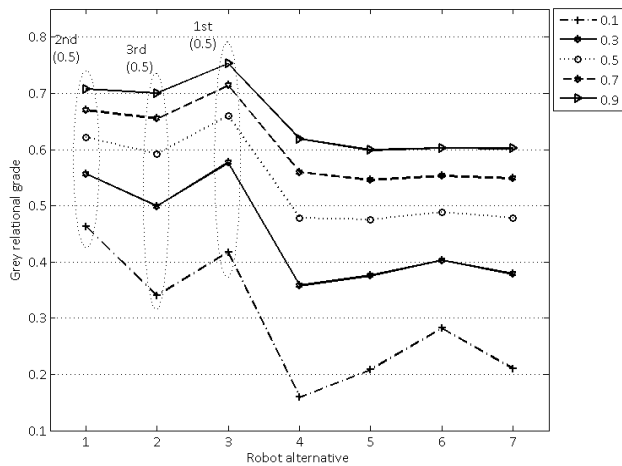


Figure 2. The impact of distinguishing coefficient on the results of GRA in case study 1

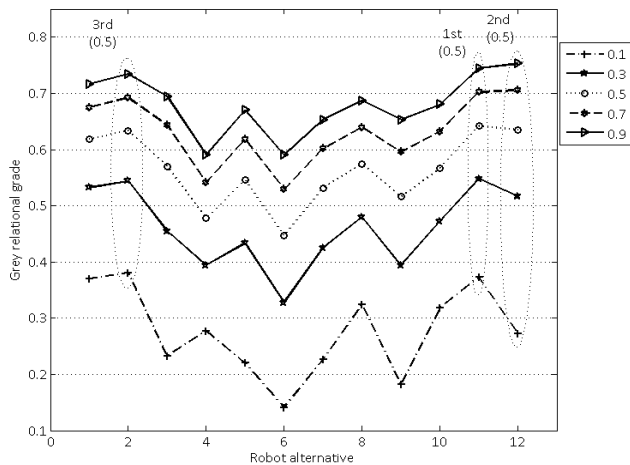


Figure 3. The impact of distinguishing coefficient on the results of GRA in case study 2

## REFERENCES

- [1] S. Mondal and S. Chakraborty, "A solution to robot selection problems using data envelopment analysis" International journal of industrial engineering computations, 2013, 4, pp. 355-372.
- [2] V. M. Athawale and S. Chakraborty, "A comparative study on the ranking performance of some multi-criteria decision-making methods for industrial robot selection", International journal of industrial engineering computations, 2011, 2, pp. 831-850.
- [3] Y. Bai and D. Wang, "Select the optimal robots and manipulators using the fuzzy multi-criteria decision making", Research journal of computation and mathematics, 2013, 1(1), pp. 1-7.
- [4] H. Haleh and K. Nezu, "A multi-criteria approach to robot selection for a computer integrated manufacturing system", 8<sup>th</sup> IEEE international conference on intelligent engineering systems, 2004, 151, pp. 569-574.
- [5] Z. Tarapata, "Selected multicriteria shortest path problems: An analysis of complexity, model and adaptation of standard algorithms", International journal applied mathematics and computer science, 2007, 17( 2), pp. 269-287.
- [6] B. Bairaga, B. Dey, B. Sarkar and S. Sanyal, "A novel multiplicative model of multi criteria analysis for robot selection", International journal on soft computing, artificial intelligence and applications, 2012, 1(3), pp. 1-9.
- [7] M. Ortmann, "Multi-criterion optimization of robot trajectories with evolutionary strategies", Electronics and energetics, 2001, 14(1), pp. 19-32.
- [8] J. Zhang, D. Wu and D. L. Olson, "The method of grey related analysis to multiple attribute decision making problems with interval numbers", Mathematical and computer modelling, 2005, 1, pp. 1-8.
- [9] Y. Zhang, "Research on multi objective shortest path problem based on grey system theory", International conference on electrical and computer engineering advances in biomedical engineering, 2012, 11, pp. 266-271.
- [10] M. Sarucan, M. E. Baysal, C. Kahraman and O. Engin, "A hierarchy grey relational analysis for selecting the renewable electricity generation technologies", Proceedings of the world congress on engineering, 2011, 2, pp. 1-6.
- [11] H. W. Lin, P. Y. Hsu and G. J. Sheen, "A fuzzy-based decision-making procedure for data warehouse system selection", Experts systems with application, 2007, 32, pp. 939-953.
- [12] Y. Kuo, T. Yang and G. W. Huang, "The use of grey relational analysis in solving multiple attribute decision-making problems", Computers and industrial engineering, 2008, 55(1), pp. 80-93.
- [13] E. E. Karsak, "Choquet integral-based decision making approach for robot selection", 2005, KES, LNAI 3682, pp. 635-641.
- [14] A. Alinezhad and R. K. Mavi, "Practical common weights goal programming approach for technology selection", Mathematical sciences, 2009, 3( 2), pp. 201-212.
- [15] V. M. Athawale, P. Chatterjee and S. Chakraborty, "Selection of industrial robots using compromise ranking method", International conference on industrial engineering and operations management, 2010, pp. 1-5.
- [16] R. K. Mavi, A. Makui, S. Fazli and A. Alinezhad , "Practical common weights compromise solution for technology selection", International journal for procurement management, 2010, 3( 2), pp. 214-230.