Hybrid Method Segmentation for Medical Image Based on DWT, FCM and HMRF-EM

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Abstract—**Segmentation is fundamental and crucial operation which comes prior to any other operation systems on image processing. We present in this paper a hybrid segmentation method of MRI to aid diagnosis of brain tumors. Our approach is based on the theory of fuzzy subsets and probabilistic models. We are proposing to obtain a tag map to initialize the class number for the Fuzzy C-Means (FCM) algorithm using wavelet transform decomposition. The Fuzzy C-Means algorithm is used as a classification phase; the last step is algorithm of Markov fields which is used as a phase of adjustment to find best partition obtained during the classification step. All of this increases the robustness of our approach to noises and defects specific to MRI images. Finally, we compare our approach to classical segmentation algorithms: Fuzzy C-Means and Markov fields. The proposed approach provides better results with a segmentation error margin 20.15% against 28.37% for Markov fields and 31.33% for Fuzzy C-Means**

Keywords-MRI, Wavelet Transform, Fuzzy C-meansn, Markov Radom Field

I. INTRODUCTION

The magnetic resonance imaging (MRI) has been booming recent years. The imagery has become a tool increasingly important in brain medicine or research in cognitive neuroscience. For diagnosing diseases related to internal brain injury and to study brain diseases, the doctor must analyze medical images. To study evolution of a tumor, it is necessary to know accurately the changes in the images. The visual interpretation of cerebral MRI is not always safe. It is for this reason that the need for automatic interpretation has been felt, to assist doctors in decision making.

In this paper, we propose to develop new approach to the diagnosis of brain tumors. In this context, our interest is concerned one of the critical steps visual processing: image segmentation. This process allows engendering a compact description of the image [1]. Many methods have been developed to automate the process of segmentation of cerebral tumors [2]. Among these approaches segmentation, we denote approaches detecting discontinuity (contour or border) [3], detection of similarity approaches (regions) [4] and approaches "hybrid" [5]. In the segmentation contours oriented, we consider that extracting primitives are lines of contrast between regions with different levels of gray and relatively homogeneous. In practice, it comes to know the areas of transition and localize the border between the regions [6]. Different methods have been proposed: the low-level methods which attach only to highlight contours, they do not include any semantic or topological knowledge, they are not adapted in the presence of noise, but they are in many approaches [1]; the methods by deformable models and level set methods which attach to change a contour initially set to the borders of objects considered, etc.. [3].

The oriented segmentation is focused on extracting regions considering their homogeneity with respect to relevant characteristics of pixels (intensity, texture ...). In this approach, various methods were developed: parametric algorithms of Mean-Shift [2] which are based on estimate the gradient of the probability density, but heavy in segmentation times; classification methods used to group objects into classes or more homogeneous group. We note: supervised approaches such as neural networks, etc… [7]; and unsupervised approaches, such as probabilistic approaches [8]. We also find the K-mean algorithms and fuzzy c-mean (FCM) for segmentation [8]. Many approaches were proposed to improve FCM algorithm like Zouaoui et al. [9], who introduced a term of spatial regularization inspired from probabilistic modeling.

Recently, to improve the robustness of the algorithm, Markov field (MRF) has been widely applied for image segmentation to take into account the mutual influences of neighboring pixels. MRF is not a method but a statistical model that can be used within segmentation methods. MRFs are often incorporated into clustering segmentation algorithms such as the K -means algorithm under a Bayesian prior model. The segmentation is then obtained by maximizing “a posteriori” probability of the segmentation given the image data using iterative methods such as iterated conditional modes or simulated annealing.

Hybrid approaches combine approaches oriented regions and contours [10]. [5] presents a method for segmentation

Probabilistic Fuzzy c-Means Expectation Maximization (PFCM-EM). Among segmentation papers, we can mention works of Sherrer and Bricq [11, 12]. This paper is organized as follows, in Section 2, we present methods used for segmentation; in section 3 we present the proposed hybrid method. The results are presented in Section 4 followed by a conclusion.

II. METHODS USED FOR SEGMENTATION

A. Wavelet Transform

Wavelet transform analyses signals in both time and frequency domain simultaneously. It is a signal analysis tool that provides a multi-resolution decomposition of an image in a bi orthogonal basis and results in a non-redundant image representation. These bases are called wavelets, and they are functions generated from one single function, called mother wavelet, by dilations and translations [13].

Discrete wavelet transform (DWT) is a multi-resolution / multi-frequency representation. It allows you to efficiency analyze signals that combine phenomena of quite different scales. Transforms steps are hierarchical filtering. This gives sub-bands image decomposition with different filters (low pass h and high pass g). This requires using a separable two-dimensional DWT (rows + columns). The input image is decomposed at each time in four sub-images (approximated image CA, DH horizontal detail, vertical detail and diagonal) [14]. Wavelet transform do not realize segmentation, it provides a label card to initialize FCM algorithm.

B. Fuzzy C-Means

Fuzzy C-Means is a fuzzy classification algorithm unsupervised. The Fuzzy C-Means algorithm is based on research of values taken by the centroids of classes and degrees of pixels belonging to these classes; which within the constraints minimize the following objective function [15] with respect to each fuzzy membership degree u_{ik} and v_i: prototype:

$$J_m = \sum_{i=1}^{c} \sum_{k=1}^{n} U_{ik}^m d^2 (x_k; v_i) \quad (1)$$

c is the number of classes, *n* is the size of the data vector (number of pixels to classify), u_{ik} is the degree of membership from pixel x_k at class *i* known by its centroid v_i, d is the degree of similarity (Euclidean distance) and *m* is the degree of fuzziness (*m > 1*). The role of index *m* is to control the contribution of noise in the data. The degrees of membership u_{ik} and centroid v_i are expressed as follows:

$$U_{ik} = \left[\sum_{k=1}^{c} \left(\frac{d^2(x_k; v_i)}{d^2(x_k; v_k)} \right)^{2(m-1)} \right]^{-1} \quad (2)$$

$$\forall_{i \in \{1,c\}} \quad V_i = \frac{\sum_{k=1}^{n} U_{ik}^m x_k}{\sum_{k=1}^{n} U_{ik}^m} \quad (3)$$

The membership values constitute a matrix to lines c (one line for each class form) and n columns (one column for each individual to classify):

$$U = [u_{ik}]_{i=1.....n} \quad (4)$$

The FCM algorithm is described below [14]:

Let $x^i = (x_1, x_2, \ldots\ldots)$ vectors to classify;

- Set parameters: *m*, *c* and ε which is the stopping criterion;
- Initialize the vector *V* by the selected number of class;
- Calculate the matrix *U* of size $(c \times n)$ by the equation (2);
- Calculate the new center of each class by using the equation (3);
- Update matrix *U* and increment the counter *t*;
- Calculate distance between the old and new center by:

$$h = \left\| V^{t-1} - V^t \right\| \quad (5)$$

- Repeat steps 2-5 as h > ε

The decision of a pixel belonging to a class is taken at the end of convergence.

C. Segmentation by Markovian modeling

Idea of Markovian segmentation is to introduce spatial interactions between labels. The Markovian model considers that the dependence of the state of a site in relation to information contained in all sites is reduced to local information contained in a vicinity of the site [16].

The observed data *(Y)* in the space of interpretation are described in order to obtain an image of label that is denoted *X*. In the framework of statistical modeling, obtaining X poses the optimization problem of determining the best performance among all possible interpretations. If X_K is an interpretation then, the desired result is given by the formula [16]:

$$X = \arg\max \{ P(X_k / Y) \} \quad (6)$$

C1. Field Markov and Gibbs distribution

A random field *Z* is a Markov random field on *V* if joint distribution *p(Z)* satisfies two properties:

$$\forall_Z \quad p(Z_i \backslash Z_{V\backslash\{i\}}) = p(Z_i \backslash Z_{N(i)}) \tag{7}$$

$$\forall_Z \quad p(Z) > 0 \tag{8}$$

Where $V\backslash\{i\}$ denotes the set of sites V private to site i, and Z_A denotes a realization of the field Z restricted to elements of A.

This random field Z is governed by a Gibbs distribution if joint probability distribution is of the form [16]:

$$p(Z) = W^{-1} exp(-U(Z)) \tag{9}$$

Where $W = \sum_z \exp(U(Z))$ a normalization function, also called partition function.

With:

$$U(z) = \sum_{c\epsilon C} V_{c(z_c)} \tag{10}$$

C2. Markovian segmentation

Segmentation consists in solving problem to incomplete data *(Y, X)* by calculating an estimate $\hat{x}$ of X knowing Y. The searched segmentation *(x)* is considered as the realization of X random field distribution $p(x|\Phi_x)$, which depends on certain parameters Φ_x. The searched configuration which maximizes the posteriori density is given by the Bayes rule by:

$$P(X_k/Y) = \frac{P(Y/X_k) \times P(X_k)}{P(Y)} \tag{11}$$

P(Y) is a constant to be maximize

$$p(x|y,\Phi) \propto p(y|x,\Phi_y) p(x,\Phi_x) \tag{12}$$

With $\Phi = (\Phi_y, \Phi_x)$.This density represents the probability of the realization x knowing the image y and Φ parameters. To solve this problem, some models are used.

According generative models, likelihood can be written as the product of local likelihoods [16]:

$$P(Y/X_k) = \prod_S P(y_k / x_{k,s}) = \prod_{i=1,\dots,I} \prod_{j=1\dots J} P(y_{i,j} / x_{i,j}) \tag{13}$$

These local likelihoods are modeled by Gaussian mixtures [12]. For monomodal segmentation $(y_i \in \mathbb{R})$, model parameters are $\Phi_y = \{\mu_k, \sigma_k, k = 1 \dots . K\}$and:

$$P(y_i / x_i = e_k, \Phi_y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu_I{}^2)}{2\sigma_I{}^2}\right) \tag{14}$$

Probability of interpretation is given by [18] by:

$$P(X_k) = \frac{\exp(-\sum_{c\epsilon C} V_c(x_k))}{\sum_i \exp(-\sum_{c\epsilon C} V_c(x_i))} \tag{15}$$

Using Bayes' rule, conditional field Y given X is a Markov random field energy function U. Thus:

$$\begin{aligned} p(x|y,\Phi) &= p(x,\Phi_X)\, p(y|x,\Phi_y) \\ &= W_{\Phi_X}^{-1} \exp\left(-U(x|\Phi_X) \lim_{x\to\infty} \prod_{i\in v} p(y_i|x_i,\Phi_y)\right) \\ &= W_{\Phi_X}^{-1} \exp\left(-U(x|\Phi_X)\right) \exp\left\{\sum_{i\in v} \log p(y_i|x_i,\Phi_y)\right\} \\ &= W_{\Phi_X}^{-1} \exp\left\{-U(x|\Phi_X) + \sum_{i\in v} \log p(y_i|x_i,\Phi_y)\right\} \end{aligned} \tag{16}$$

$p(x|y,\Phi)$ is Gibbs distribution, which corresponds to a MRF whose energy function [17]:

$$U(x|y,\Phi) = U(x|\Phi_x) - \sum_{i\epsilon v} \log p(y_i|x_i,\Phi_y) \tag{17}$$

With $\Phi = (\Phi_x, \Phi_y)$. Energy function of Equation (17) is a central function in segmentation models Markov. It is composed of two terms. The second term represents the data attachment term and U is a regularization term reflecting spatial correlation.

The segmented image must be a configuration which maximizes the posterior probability in the sense of a certain criteria. The most common test is the Maximum A Posteriori (MAP). It comes to solve the equation (6), which is written as [16]:

$$\hat{x} = argmin\{U(Y/X, \Phi) + U(X)\} \tag{18}$$

The maximum of label should minimize forms of energy. The probable energy is given by:

$$U(X \setminus Y, \Phi) = \sum_i U(y_i \setminus x_i, \Phi) \tag{19}$$

$$U(X \setminus Y, \Phi) = \sum_i \left[\frac{(y_i - \mu_{xi})^2}{2\sigma_{xi}{}^2} - ln\sqrt{2\pi}\sigma_{xi}\right] \tag{20}$$

Energy $U(X)$ is given by:

$$U(X) = \sum_{c\epsilon C} V_c(x)) \tag{21}$$

$V_c(x)$ is the potential on cliques.

The iterative algorithm for computing $\hat{x}$ by MAP is given according to [18]:

- Initialize $X^{(0)}$, provided in loop EM of algorithm (Expectation Maximization)
- Give $X^{(0)}$; for all $1 \le i \le N$, search:

$$x_i^{(k+1)} = \arg\max_{k \in L} \left\{ U\left(y_i \middle| l\right) + \sum_{j \in N_i} V_c\left(l, x_j^{(k)}\right) \right\} \quad (22)$$

- Repeat step 2 as a $U\left(Y/X,\theta\right)+U\left(X\right)$ does not converge to maximum of k

$l = \left(x_i, \dots \dots, x_l\right)$ is space of tags.

Expectation Maximization is an algorithm which aims to calculate parameter estimation by maximum likelihood in a model with incomplete data *(X, Y)*. It seeks to estimate parameters for which observed data are most likely. The EM algorithm is an iterative maximization of *Q* function defined in *r* iteration:

$$Q(\Phi|\Phi^{(t)}) = E[\ln P(X, Y|\theta)|Y, \theta^{(t)}]$$

$$= \sum_{x \in X} P(X|Y, \theta^{(t)}) \ln P(X, Y|\theta) \quad (23)$$

Idea is to maximize each at iteration of successive approximations of local likelihood. According to [12], EM algorithm is:

1. Initializing the first value $\Phi^{(0)}$ of Φ
2. Iterative minimization of $\Phi^{(t)}$

- Step *E* (Expectation) calculate conditional expectation $Q(\Phi \middle| \Phi^{(t)})$;
- Step *M* (Maximization): Updating parameters for next estimate $\Phi^{(t+1)} = \arg\max_{\Phi} Q\left(\Phi \middle| \Phi^{(t)}\right)$. As $\Phi^{(t+1)} \to \Phi^{(t)}$, repeat step *E*.

C3. Choice of model

A Potts model of four neighboring takes into account distance between the pixels in terms of spatial correlation was seen. In this case, potential of pair's pixels of a clique is:

$$V_c\left(x_i, x_j\right) = \frac{1}{2}\left(1 - I_{x_i, x_j}\right) \quad (24)$$

With $I_{x_i, x_j} = \begin{cases} 0 & si\ x_i \neq x_j \\ 1 & otherwise \end{cases}$

The probability densities are Gaussian considered for each label, such that:

$$P(y_i|x_i = e_k, \Phi_y) = G_k(y_i, \Phi_l) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu_l^2)}{2\sigma_l^2}\right) \quad (25)$$

The density *x* of a posteriori knowing the observations *y* and parameters Φ is then written:

$$P(X|Y, \Phi) = W_{\Phi_x}{}^{-1} \exp(-U(X|Y), \Phi) \quad (26)$$

Segmentation is performed on the principle of maximum a posteriori (MAP) maximizing the density. The maximum requires parameter estimation θ which will be obtained by EM algorithm.

The *Hidden Markov Radom Field- Expectation-Maximization* (HMRF-EM) is used for segmentation of MRI image [19, 20]. It uses the K-means to initialize labels grayscale pixel. The initial segmentation gives label $X^{(0)}$ by MAP and the initial parameters by EM. The result is segmentation by hidden Markov random field described by HMRF-EM. The HMRF-EM algorithm is:

- Choice of parameters $\theta^{(0)}$;
- Calculate the probability distribution $P^{(t)}\left(y_i \middle| x_i, \theta_{x_i}\right)$;
- Use parameters $\theta^{(t)}$ to estimate MAP labels according to equation (18);
- Calculate posteriori distribution for all configurations $l \in L$ and all y_i pixels. Namely:

$$P^{(t)}(l|y_i) = \frac{G_k(y_i, \theta_l) P\left(l \middle| x_{N_i}^{(t)}\right)}{P^{(t)}(y_i)} \quad (27)$$

$x_{N_i}^{(t)}$ is the configuration of neighborhood $x_i^{(t)}$

$$P^{(t)}(y_i) = \sum_{l \in L} G_k(y_i, \theta_l) P\left(l \middle| x_{N_i}^{(t)}\right) \quad (28)$$

and

$$P\left(l \middle| x_{N_i}^{(t)}\right) = \frac{1}{z} exp\left(-\sum_{j \in N_i} V_c\left(l, x_j^{(t)}\right)\right) \quad (29)$$

- Use $P^{(t)}(l|y_i)$ for update settings:

$$\mu_l^{(t+1)} = \frac{\sum_i P^{(t)}(l|y_i) y_i}{\sum_i P^{(t)}(l|y_i)} \quad (30)$$

$$\left(\sigma_l^{(t+1)}\right)^2 = \frac{\sum_i P^{(t)}(l|y_i)\left(y_i-\mu_l^{(t+1)}\right)^2}{\sum_i P^{(t)}(l|y_i)} \quad (31)$$

III. Proposed Hybrid Method

A. *Approach description*

Our approach is an automatic approach based on the theory of fuzzy sets perfectly adopted for handling uncertain and imprecise characteristics of MRI data. We proposed to combine three methods: DWT, FCM and HMRF-EM. FCM is used for classification phase (label map). This phase used to initialize the centroids. It was proposed to include in modeling, of contextual data through the use of Markov fields; which increases robustness of the approach proposed to address the noise and artefacts specific MRI. HMRF-EM is used after the clustering algorithm (FCM), which provides an initial partition to hidden Markov fields. This improves partition obtained as a result of the classification step. Figure 1 provides a global description of the proposed approach.

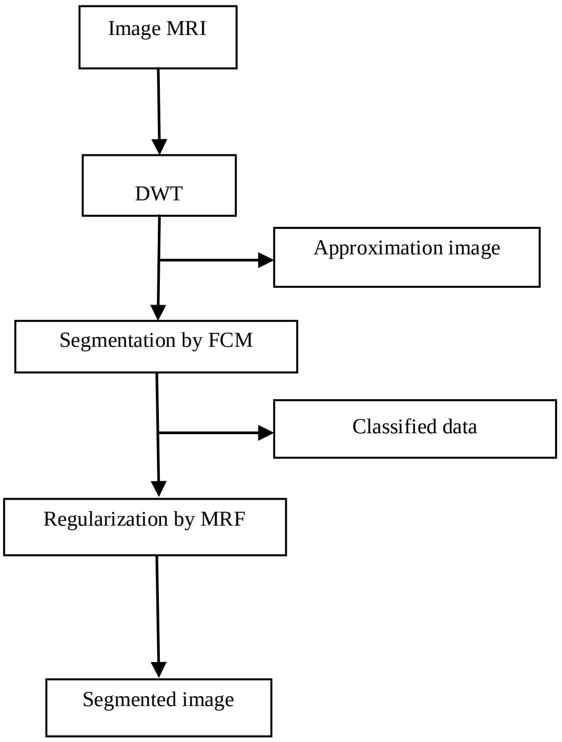


Figure 1. Global description of our method

B. *Algorithm of proposed approach*

Our segmentation algorithm is based on FCM where its flaws are improved. It consists of three steps. The first is algorithm of wavelet transform which aims has given a label map for the number of classes FCM, second step is FCM algorithm, which is classification phase and the last step is HMRF-EM algorithm for phase regularization; i.e. find the better partition in the classification step.

Algorithm is as follows:

Input: either image $X^i = (X_1, X_2, \ldots\ldots)$

Step 1: Wavelet decomposition to the resolution level j;

Step 2: At resolution j, fuzzy classification;

- Set parameters m; c; ε;
- Initialize the vector *v* by *c* randomly chosen center;
- Calculate matrix U by equation (2);
- Calculate the new of class center by equation (3);
- Update matrix U and the counter t;
- Calculate the distance between new and old centers by equation (5);
- Repeat previous steps up to convergence (h> ε.)

Step 3: Regularization by HMRF-EM

- Estimate the parameters $\theta^{(0)}$ by K-means algorithm with k = 2;
- Calculate the probability distribution $P^{(t)}\left(y_i \middle| x_i, \theta_{x_i}\right)$;
- Estimate the labels by MAP according to equation (18), using $\theta^{(t)}$ parameters;
- Calculate posteriori distribution for all configurations $l \in L$ and all pixels y_i by equation (27);
- Use the posteriori distribution for updating the parameters μ_l and σ_l according to equations (30) and (31).

Output: segmented image.

IV. Results and Discussion

For different algorithms, we used Internet Brain Segmentation Repository [21]. Like parameters used, we have the class number of image (c = 2) and the stop parameter (ε = 0.001). The iteration count is adjusted according to images.

Noise is another defect characterizing the IRM images. Its effect can be modeled by FCM, but this modeling depends heavily on fuzziness index m. For our work, the index m = 2 was retained. This choice was made in relation to work Laguel [22], on the influence of blur factor on the results of classification for cerebral MRI.

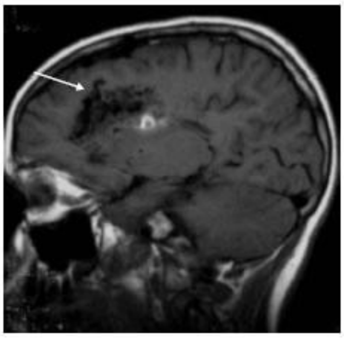
The FCM algorithm is applied to the weighted MRI T2 of size 392 × 317. Figure 2 is the result obtained by FCM algorithm. Segmentation method requires very few parameters, which is an important advantage.

However, FCM algorithm does not provide a good segmentation of cerebral MRI, when used alone. This algorithm takes into account global image information without considering the pixel as belonging to two-dimensional image.

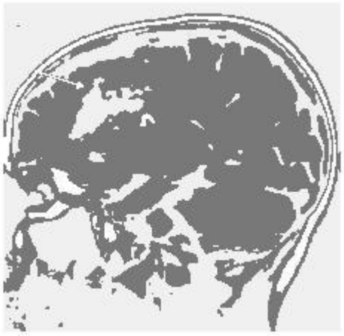
a)

b)

Figure 2. a) Original image; b) Segmented image by FCM

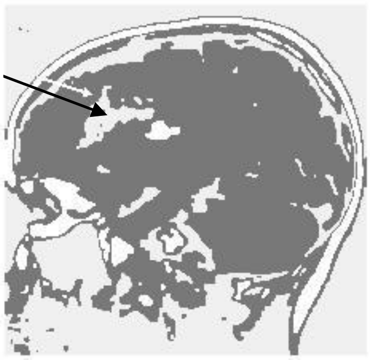


a)

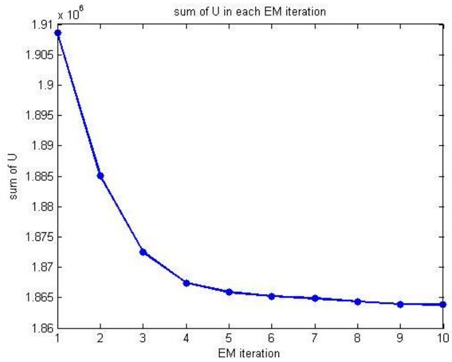


b)

Segmented tumor



c)



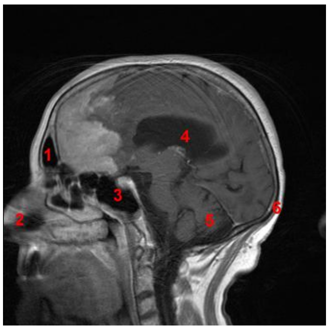
d)

Figure 3. a) Original image; b) Initial label obtained by K-means for k = 3; c) Final segmentation obtained by HMRF-EM; d) Variation of energy a posterior according of iterations EM algorithm

Analysis by Markov field described by HMRF-EM algorithm was used for the segmentation of MRI images. K-means is used to initialize labels grayscale pixel. The initial segmentation gives label $X^{(0)}$ by MAP and the initial parameters by EM. As segmentation parameter, we have a maximum iteration of MAP = 10, maximum iteration EM = 10 and a number labels (k = 3) for initialization K-means. The HMRF-EM algorithm was applied on a T1 weighted MRI image of size 240 × 229. Figure 3 shows the result obtained after segmentation with time 193 seconds.

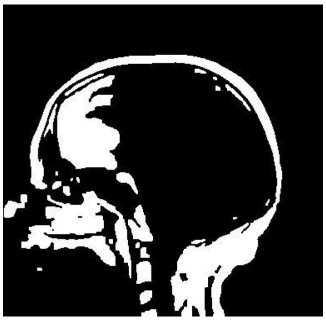
Starting from an initialization map obtained by K-means (Figure 3b) and after 10 iterations, we obtain the segmented image (Figure 3c). Figure 3d shows that the condition of segmentation by MAP is respected, ie a maximum iteration for minimum of energy: to 10 iterations,energy approaches $1{,}863 \times 10^6$ which is the minimum energy for iteration maximum.

To validate our approach, a comparison of different approaches segmentation is made. Figure 4 shows a frontal meningioma MRI for different segmentation methods.



Original image

Segmentation by fuzzy-c means



Segmentation by HMRF-EM

Segmentation by the proposed approach

Figure 4. Result of comparison between different approaches segmentation

TABLE I. COMPARISON OF DIFFERENT METHODS

Images	size (pixels)	Different methods	Time (S)	Area (pixels)	Area given by radiologist (pixels)	SE
	147456	FCM	12,66	190		1,42
A	147456	HMRF-EM	251,17	169		1,27
	147456	Proposed approach	270	126	133	0,94
	58000	FCM	3,57	55		1,38
B	58000	HMRF-EM	100,52	48	41	1,17
	58000	Proposed approach	265,12	36		0,87
	54960	FCM	2,45	40		0,82
C	54960	HMRF-EM	92, 974	31	50	0,62
	54960	Proposed approach	100,23	27		0,54

Table 1 shows a comparison of results. The surface is calculated to examine the robustness of proposed approach. The surfaces of regions detected by proposed approach are more accurate as compared to other methods, and those given by radiologist. The noise sensitivity (SE) of different methods was evaluated by the formula [16]:

$$SE = \frac{TP}{TP + FN} \tag{32}$$

This sensitivity (*SE*) corresponds to proportion of true positive relative to all structures that should be segmented. *TP*: true positive and *FN*: false negatives

Table 2 illustrates the comparison of the average error margin each algorithm. The proposed approach has a margin of error 20.15% to detect interest areas compared to other methods and radiologist.

TABLE II. COMPARISON OF MARGINS ERROR

different methods	Average margin of error (%)
FCM	31,33
HRMF-EM	28,37
Proposed approach	20,15

Figure 5 shows sensitivity to noise of different methods depending on image size. Our approach has smallest sensitivity, and shows better strength compared the other two methods.

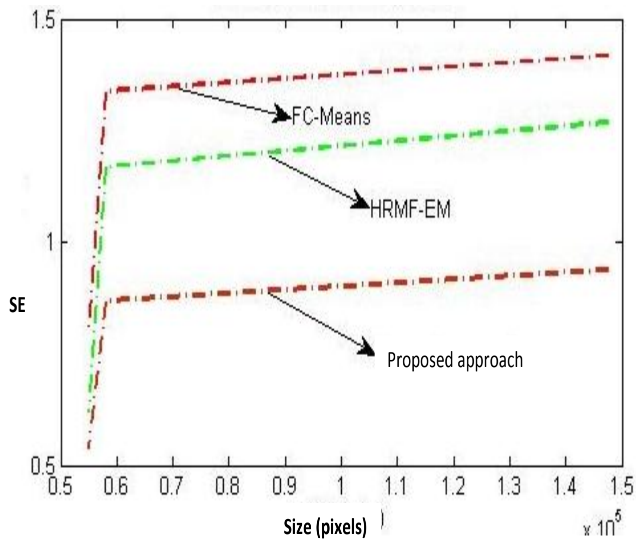


Figure 5. Variation of sensitivity according to image size.

Our segmentation approach is compared to FCM algorithm, HMRF-EM and gives satisfactory results, by calculating the area of tumor segmented by each method. From Table 1, for one image size of 147456 pixels, we have an area of 126 pixels for proposed approach, 190 pixels for FCM and 169 pixels for HMRF-EM algorithm. With same image, the segmentation error is 5.26% for proposed approach, 27.06% for HRMF-EM algorithm and 42.85% for FCM.

V. CONCLUSION

This paper presented a hybrid medical image segmentation technique. Experimental results shown in this paper confirm the utility of the proposed model. The obtained results by segmentation are more interesting. The proposed approach allows a better modeling of brain and these tumors, with fuzzy and probabilistic theory. Performance and validation the new approach to segmentation cerebral pathologies are demonstrated through MRI images based GEsystems data. The results obtained give advantage to our approach compared to FCM algorithm, HMRF-EM. Our approach shows robustness compared to conventional algorithms. However, it shows the interest of proposed approach. It appears that these methods can not replace the eye of radiologist; they are a reliable tool of aid to diagnostic.

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