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Generating Grid DTM Based on Altitude Point System

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Abstract — **Geographical information systems (GIS) are means of effective collection, storage, searching, transformation, analysis and display of geographic information. Geographic data are spatial data on the geographic sphere (the landscape). Their most important feature is the aspect of position, recorded, for example, as a digital terrain model. The basic requirement set up in terms of the terrain model is to calculate the terrain altitude of a specific location, object, etc. The ways of fulfilling this requirement are related mostly to the terrain model type. This article describes the methods of creation of the contour and slope lines based on a terrain model. It describes an algorithm to produce terrain models from the measured altitude, with the possibility of displaying contour lines and slope profiles. The altitude point system ensures that we interpret the system of contour lines and slope profiles correctly and it provides a simple interface to construct contour lines, slope profiles and the digital terrain model.**

Keywords - contour lines, geographical information system, slope lines, terrain model

I. INTRODUCTION

Gathering data on the position of important objects and resources on the surface of the Earth was always a significant activity of mankind. With various motives, there was always a close correlation with astronomy; man made use of the relations of the Earth and bodies of Cosmos. The activities of man brought ever newer knowledge on the spatial objects on the Earth. The results of these efforts were maps, such as astronomical, geological and tectonic maps, soil maps, demographical, traffic and climate maps, etc. Unlike the original need of specifying the exact position of an object, now we have to find ways of storing ever growing quantities of data regarding these objects.

Geographic data are spatial data on the geographic sphere (the landscape). Their most important feature is the aspect of position. Further properties are the variability of the data over time and the quality or quantity attribute or characteristic describing the given geographical object or phenomenon. Not less than 80% of all information is position-related, therefore it can be processed by means of a geographical information system. Geographical information systems (GIS) are means of effective collection, storage, searching, transformation, analysis and display of geographic data.

Geographical information systems are a set of hardware and software assets to store, process and use geographical information in two forms: as graphics and as data, mutually connected, classified by type. As a computerized tool to map and analyze the objects and phenomena of the real world, it combines common database operations, such as information storage and management, statistical calculations with the unique capabilities of spatial analysis display, provided by cartographic maps [1].

Cartography, dealing with the display and study of spatial positions and mutual correlations of natural phenomena and society uses graphical and modeling tools to generate maps. Some of the information serving as cartographic inputs is provided by geodesy. The results of many geodetic measurements serve as inputs for processing maps of different kinds, serving different needs [2], [3].

II. MAPS

When drawing maps, one has to reduce the image of reality. With each reduction we generalize the real world – exclude meaningless facts and details from the projection of the real world on the map (Fig. 1). This generalization must not affect the legibility, the illustrative nature and the aesthetics of the map.

When producing map content, we use the conventional cartographic means of expression (cartographic symbols), such as lines (e.g. contour lines), shape signs (geometrical, illustrative symbols, letters, etc.), lines of various types, numerical data displayed in various colors, area symbols, coloring of surfaces, shading, rasters, points, movement signs and many others. In the various types of automated processing, digital maps are gaining ever more space.

A digital terrain model (DTM) is a set of selected points – topographical surface points. The terrain surface is very diverse. In the nodes of the digital terrain model it is defined, while the elevation of the remaining points has to be appropriately interpolated. As to the approximation of the topographical surface, terrain points may be regular (they can be defined as tangent planes to the topographical surface) and singular (no tangent planes may be defined to them,

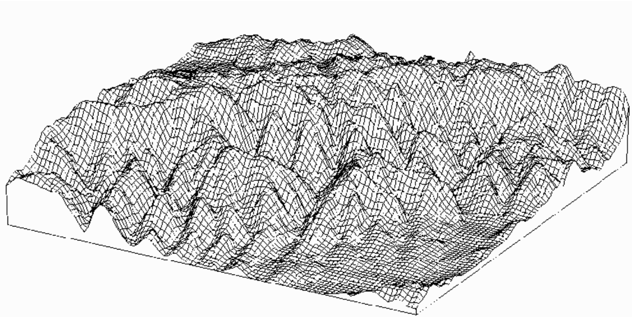

Figure 1. Axonometric, wire-frame terrain display

these nodes form "terrain edges", such as the nodes of embankment or hole edges, etc.).

When creating DTMs, we mostly use isolated elevation points (an array of discrete points) with varying density and layout. When estimating values for which no data are available and the subsequent generation of DTMs we use various spatial interpolation methods. When selecting an interpolation method one has to take multiple factors into account, such as the kind of the interpolation phenomenon, the nature of the surface (the vertical and horizontal configuration, terrain edges, etc.) or the aim of the DTM. Selecting the optimal interpolation method is mostly a matter of taste and it may greatly influence the precision of the final elevation values of the final DTM.

The various methods of spatial interpolation are described in the available literature [4]:

1. Inverse Distance Weighting;
2. Simple Spatial Regression;
3. Kriging;
4. Splines.

A. Inverse Distance Wweighting

Inverse distance weighting (IDW) [5], [6] is used to specify the elevations of the grid cells using a weighted average. The "z" interpolated cell elevation value is calculated using the elevation values of the points situated in a specified distance from the center of the cell. IDW – as a local interpolation method – acts as a filtering window, calculating the average value of from the points in the vicinity. The search radius defines the points, which will be included into the interpolation process.

B. Simple Spatial Regression / Trend Surfaces

Having a continuous feature in space one may calculate the points of the interpolated surface with a polynomial function, i.e. a trend. Trend interpolation adapts the surface to a set of points using manifold (polynomial) regression. The best coefficients of a given nth order polynomial are to be selected by the method of least squares. The surface may be a plane (linear regression model - 1st order polynomial) or a surface of a complex object (higher order polynomials). The final surface does not pass through any of the input points. By increasing the polynomial order we may replicate more complex surfaces and reduce randomness. However, in this case the result is more error-prone (and we get greater differences) on the edges of the territory or at the areas beyond measurement.

C. Kriging

Kriging [5], [6] is a geostatic method. We expect the neighboring points being auto-correlated in space. The interpolated surface consists of three elements: the drift (a general trend of the surface, depending on the change of coordinates), a regionalized change (fluctuation, the essence of which cannot be specified as a mathematical function, but as some spatial correlation) and random noise (differences not spatially correlated, thus they cannot be summed up). These elements are defined by means of variograms, which provide the quantification of correlation of two arbitrary variables. This quantification is used in kriging to survey and apply the best interpolation procedures. Kriging is an exact interpolation method and – unless the noise rate is too large – it provides very precise results. On the other hand, it requires a lot of computing.

D. Splines

The method of splines uses mathematically defined curves [7] which partially interpolate the individual parts of the surface. The curvature of the resulting surface is minimal. To interpolate surfaces one would use so-called bi-cubic splines – regular (these result in smoother surfaces) or tight (these produce rougher surfaces, though adjacent to the input points). The advantage of this method is that one may modify only parts of the terrain, without the necessity of recalculating the whole surface. However, the disadvantage is that the resulting relief is very smooth, since any barriers or jumps are smoothed out. It works best when interpolating very smooth surfaces, such as depictions of climate phenomena. It is often used to smooth surfaces.

If the general form of the equation describing a continuous topographic surface in a limited area is the following:

$$z = f(x, y)\,. \qquad (1)$$

Having an unknown analytical expression and unknown coefficients, this surface may only be defined by means of a final set of *n* nodes appropriately laid out on the surface of the area, the *N*-dimensional vector of coordinates of which is the following – known from measurements:

$$\vec{z}_{n,l} = \vec{f}_{n,l}(x, y)\,. \qquad (2)$$

The elevations of the further points (the *z* coordinates) are specified by an *m*-order interpolation polynomial as follows, approximating the unknown expression of topographical surface (1):

$$P_m(x,y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + \ldots + \\ + a_{m0}x^m + a_{m-1,1}x^{m-1}y + + a_{0m}y^2 . \quad (3)$$

The minimum number of nodes required to specify the (3) coefficients is:

$$n_{\min} = \frac{(m+1)(m+2)}{2} . \quad (4)$$

where n is: $n > n_{min} > m$. The topographical surface may be approximated also using other interpolation polynomials, for example by B-spline polynomials or non-uniform rational b-splines.

III. Contour Lines

The basic requirement set up in terms of the terrain model is to calculate the terrain altitude of a specific location, object, etc. and to display the contour line and slope line structures. The ways of fulfilling this requirement are related mostly to the terrain model type. Except for the depiction as images we may use also the contour line projection of surfaces, using the Euler-Monge method [7]. The advantages are evident mainly in the field of technical applications. This is the method used to create geographical and meteorological maps, as well as images showing the potential of the electric field.

The use of contour lines may be extended to functions not defined in planar space but on curved surfaces [9], such as the distribution of temperature on the surface of an object having a complex shape. The function values are specified in the values next to the contour lines, the slope is specified by their density.

Let function (1) specifying the shape of the topographical surface be limited in the interval $\Omega = [\alpha,\beta]x[\gamma,\delta]$. The equation of the contour line having the λ nominal value will be as follows:

$$M(\lambda) = \{(x,y) \in \Omega \mid f(x,y) = \lambda\} . \quad (5)$$

Thus the above contour line (5) is a set of points, where the function value is λ. If the respective contour lines do not have their nominal values specified, first we calculate the extremes (minimum and maximum values) of the function in the Ω interval:

$$f_{\max} = \max_{\Omega}\{f(x,y)\} \\ f_{\min} = \min_{\Omega}\{f(x,y)\} . \quad (6)$$

In practice, we may use a non-linear optimizing algorithm for this purpose, returning the extremes of the function in question. The calculated points may be then used even in the display of the contour line structure. Since the resolution of monitors is relatively low, this non-linear optimization algorithm is very simple, i.e. we compute the function values for each point of the Ω interval, corresponding to the pixels of the screen. The maximum and minimum values are specified using these data.

To draw c contour lines with a constant mutual distance (Fig. 2, $c = 6$), one may calculate the nominal values of these as follows:

$$\lambda_i = f_{\min} + i \cdot \frac{f_{\max} - f_{\min}}{c+1}; \quad i \in \{0,1,\ldots,c+1\} . \quad (7)$$

The following algorithm may be used to draw contour lines:

INPUT: a list of the coordinates of the nodes of the surface: $(x_{0,0}\ y_{0,0}\ z_{0,0}), (x_{0,1}\ y_{0,1}\ z_{0,1}), \ldots, (x_{n,l}\ y_{n,l}\ z_{n,l})$.

1. Specify the elevations of the further points (the z coordinates) by means of an m-order interpolation polynomial, as specified in (3).
2. Calculate extremities in accordance with (6).
3. May the area Ω be represented by the $[X_0, Y_0] \times [X_p, Y_q]$ rectangle on the screen. We split the rectangle so that each line contains p pixels and each column q pixels:

$$p = X_p - X_0, \quad q = Y_0 - Y_q . \quad (8)$$

4. The correlation between the (x_r, y_s) points of the Ω area and the pixels (X_r, Y_s) on the screen may be specified using the following equations:

$$\begin{aligned} x_r &= \alpha + \frac{r}{p}(\beta - \alpha) \rightarrow \\ X_r &= X_0 + r; \quad r \in \{0,1,\ldots,p\} \\ y_s &= \gamma + \frac{s}{q}(\delta - \gamma) \rightarrow \\ Y_s &= Y_0 + s; \quad s \in \{0,1,\ldots,q\} \end{aligned} . \quad (9)$$

5. In each point, where the function

$$g_i(r,s) = f(x_r, y_s) - \lambda_i; \quad i \in \{0,1,\ldots,c\} . \quad (10)$$

changes its sign we have to change the color of the point of the raster.

IV. Slope Lines

The traditional expression of elevation uses contour lines (i.e. the contour line model), with some significant elevation points added. However, this expression is not continuous, because the contour lines represent only selected elevations (the basic interval of contour lines). To describe the terrain, we need also vertical profiles (slope profiles), making the terrain model more precise. An important feature of contour lines and slope profiles is that they are mutually perpendicular. Two contour lines or two slope profiles never cross, by definition.

The gradient of function (1) is a vector, the coordinates of which are the partial derivations of function *f*, thus:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}(x,y), \frac{\partial f}{\partial y}(x,y) \right). \tag{11}$$

The gradient vector points (11) always in the direction of the largest increase. Let E_x and E_y represent the coordinates of the gradient vector:

$$E_x = \frac{\partial f}{\partial x}, \quad E_y = \frac{\partial f}{\partial y}. \tag{12}$$

We approximate the unit length of the slope profile curve with a line segment collinear with the tangent and the projections of which on the axes of the O_{xy} coordinate system are d_x and d_y, i.e.:

$$\frac{d_x}{d_y} = \frac{E_x}{E_y}. \tag{13}$$

The length of the line segment (13) is given by the following equation:

$$ds = \sqrt{d_x^{\,2} + d_y^{\,2}}. \tag{14}$$

Therefore the equations of the individual projections can be put like this:

$$d_x = \frac{E_x}{E} ds, \quad d_y = \frac{E_y}{E} ds. \tag{15}$$

where E is the gradient length:

$$E = \sqrt{E_x^{\,2} + E_y^{\,2}}. \tag{16}$$

The following algorithm may be used to draw slope profiles:

1. Specify the starting point (x_k, y_k).
2. Specify the unit length of the arc (d_s).
3. Now compute the gradient vector coordinates.
4. Using (15), calculate d_x and d_y. The following applies to the coordinates of the new slope profile point:

$$\left(x^{k+1}, y^{k+1}\right) = \left(x^k + d_x, y^k + d_y\right). \tag{17}$$

5. Using a procedure written for this purpose, draw the line segment, which approximates the unit arc of the slope profile.
6. Repeat the above steps 1 to 5 also for the newly calculated points.

The shorter of the *ds* unit arc is, higher is the precision. The endpoint of the slope profile is identical to the point providing the global minimum or maximum of the function $z = f(x,y)$. In these points, we have to define the termination condition of the algorithm, because in the extremities, the partial derivations are equal to zero. The algorithm termination condition is $E < \varepsilon$, where ε is a sufficiently small positive number. Except for this, the slope profiles may end also at the edges of the specified area, therefore we have to define the termination condition also for this case.

V. A System of Elevation Points

If we do not know the analytical expression of the topographical surface, we can use the data resulting from the surveyed and/or defined elevation points. The basis of the said terrain modeling method is the following physical model:

1. The points of the surface among the nodes of the net splitting the domain of definition may move only vertically.
2. The points of the surface are connected to the $z = 0$ plane due to an elastic force:

$$F_1 = -azk. \tag{18}$$

   where z is the distance of the point from the xy plane and a is the coefficient of elasticity.
3. The elevation points (x_k, y_k, z_k) act on the points of surface having the (x, y, z) coordinates with a gravitational force, the magnitude of which is proportional to the second power of their mutual distance – the following applies to this force:

$$\vec{F}_2 = \sum_{k=1}^{n} \frac{w_k\left[(x-x_k)\vec{i} + (y-y_k)\vec{j} + (z-z_k)\vec{k}\right]}{\left[(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2\right]^{3/2}}. \tag{19}$$

   where w_k is the weight of the *k*-th elevation point.
4. The position of the point is specified by the following equilibrium of forces:

$$F_1\vec{k} + F_2\vec{k} = 0. \tag{20}$$

   if $a = 1$, this leads to the following equation:

$$z = \sum_{k=1}^{n} \frac{w_k(z_k - z)}{\left[(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2\right]^{3/2}}. \tag{21}$$

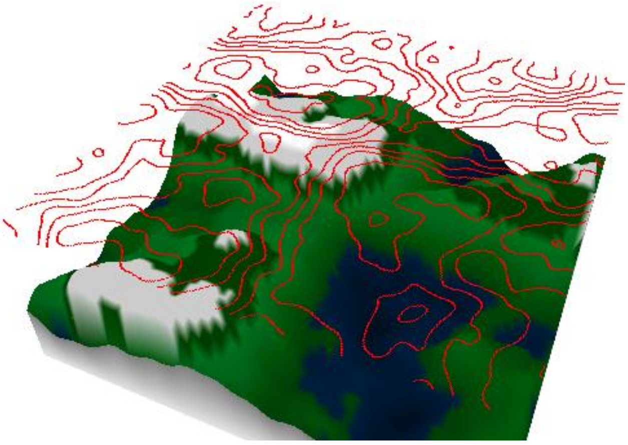

Figure 2. Digital terrain model with contour lines, c = 6

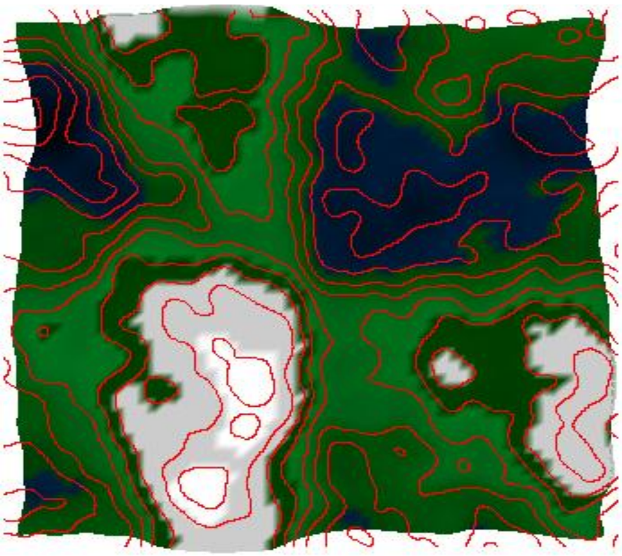

Figure 3. Digital terrain model (top view) with contour lines, c = 6

The above equation is recursive and may be calculated iteratively (starting from the starting point, where $z = 0$ we apply the (21) equation until the difference of two subsequent values is not smaller than ε, where ε is a sufficiently small positive number representing calculation errors).

The terrain and their contour lines produced with the above algorithm are displayed on Fig. 2-3.

VI. CONCLUSION

Today's novel technologies allow data acquisition with a higher level of precision, speed and regularity than ever before.

Software – being at the core of these technologies – allows the processing of raw data from various systems gathering them. Many types of geographical operations are best visualized as maps or graphs. Maps are very effective means of storing and transferring spatial information.

This article deals with the creation of contour line and slope line maps. The article contains an algorithm to draw contour lines and slope profiles, if the analytical expression of the topographical surface is at hand. If we do not know the analytical expression of the topographical surface, we can use the defined elevation points. The altitude point system ensures that we interpret the system of contour lines and slope profiles correctly. It provides a simple interface to construct contour lines, slope profiles and the terrain relief. A disadvantage of this system is that by increasing the number of elevation points the number of parameters of these structures grows, too.

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