The Dynamical Mode of Uncertain Nonholonomic Systems with Affine Kinematic Constraints

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Abstract—**In this paper, applying the elementary transformation and the D' Alembert-Lagrange principle only, we firstly establishes the dynamical model of the uncertain nonholonomic control systems with affine kinematic constraints(AKC) in the presence of both kinematic and dynamic parametric uncertainty. Th*e method which proposed in this paper can avoid the calculation of*** $v(q)\in R^{n-m}$.

Keywords- ***nonholonomic systems, dynamical model, Affine kinematic constraints, elementary transformation, D' Alembert-Lagrange principle***

I. INTRODUCTION

This A system with nonholonomic constraints is that the system can be driven to a desired configuration using fewer inputs (forces/torques of actuators) than the degrees of the freedom of the system [19] such as rolling contact, hopping robots [1], underactuated manipulators[2], space robots [3], mobile cars[4], trailers[5], acrobat robots[6] and so on, which is a notable characteristic of the system with nonholonomic constraints, a great number of research results has been produced in studies on the dynamical models of nonholonomic control systems with linear constraints due to the demand for control of the referred systems. Control of object motion and internal force by multi-fingered robot hand with rolling contact have been widely studied in control theory used to deal with uncertain dynamical systems ([7], [8], [9], [10], [11]). In [20], a three-fingered robot hand manipulating an object with rolling contact is considered, where each finger has three degrees of freedom, and the authors proposed a method to control the object motion and the internal force. In [21], the authors considered a two-fingered robot hand manipulating an object with the pure rolling contact, where each finger has six degrees of freedom. Ilya Kolmanovsky et al. [22] studied a class of nonholonomic control systems in extended power form is. Under the assumptions Lagrange's equations, it is demonstrated that can be can be transformed into the extended power form including classical nonholonomic constraints. Jian Wang et al. [23] proposed a stable motion tracking control law for mechanical systems subject to both nonholonomic and holonomic constraints, developed a control law at the dynamic level and can deal with model uncertainties, and the proposed control law ensured the desired trajectory tracking of the configuration state of the closed-loop system. Zhendong Sun et al. [24] addressed the problem of feedback stabilization of nonholonomic chained systems within the framework of nonregular feedback linearization, formulated the nonsmooth version of nonregular feedback linearization, provided a criterion for nonregular feedback linearization is, and proved that the chained form is linearizable via nonregular feedback contro.

In general, much of the research focused on the Pfaffian constraint $J^T(q)\dot{q}=0$, $J(q)\in R^{n\times m}$, and eliminating Lagrange multiplier with natural orthogonal complement technique. But this method needs $J(q)$ is precisely available, and needs to calculate the matrix $S(q)\in R^{n\times(n-m)}$, whose column vectors span $J^T(q)$ calculate a set of linearly independent vector field $v(q)\in R^{n-m}$ such that $\dot{q}=S(q)v(q)$. In this paper, applying the elementary transformation and the D' Alembert-Lagrange principle only, we derive the equations of motion of a mechanics system subjected to affine kinematic constraints in the presence of both kinematic and dynamic parametric uncertainty. The method which proposed in this paper can avoid the calculation of $v(q)\in R^{n-m}$.

II. PREPARE YOUR PAPER BEFORE STYLING SYSTEM MODEL

In this paper, we consider a mechanical system whose state is defined by generalized coordinates $q=(q_1,q_2,\cdots,q_n)^T$ and velocities $\dot{q}=(\dot{q}_1,\dot{q}_2,\cdots,\dot{q}_n)^T$.

The nonholonomic constraint that is affine in the velocities and time independent is

$$J^T(q)\dot{q}=A(q), \quad (1)$$

where $J(q)\in R^{n\times m}$, $A(q)\in R^m$, and $J(q)$ as follows:

$$J(q)=(j_1(q),j_2(q),\cdots,j_m(q))^T, \quad (2)$$

$$j_\gamma(q)=(j_{1\gamma}(q),j_{2\gamma}(q),\cdots,j_{n\gamma}(q))^T,\ \gamma=1,2,\cdots,m.$$

Owing $J(q)\in R^{n\times m}$ is of full rank, thus there exist matrices $P_1,P_2,\cdots,P_r$, such that

$$J^T(q)P_1P_2\cdots P_r=\begin{bmatrix}J_1(q) & J_2(q)\end{bmatrix}, \quad (3)$$

where $P_k\in R^{n\times n}(k=1,2,\cdots,r)$ is the matrix produced by exchanging row i and row j of the identity matrix, $J_1(q)\in R^{m\times m}$ is nonsingular. Letting

$$\bar{N}=P_1P_2\cdots P\in R^{n\times n}, \quad (4)$$

$$S(q)=\bar{N}\begin{bmatrix}J_1^{-1}(q)J_2(q)\\ -I_1\end{bmatrix}, \quad (5)$$

where $I_1\in R^{(n-m)\times(n-m)}$. It is easy to deduce that $S(q)$ is of full rank, the following relation holds:

$$J^T(q)S(q)=0. \quad (6)$$

and there exists a full rank matrix $S_1(q)\in R^{(n-m)\times n}$, which satisfies

$$S_1(q)S(q)=0 \quad (7)$$

For the sake of convenience, we define

$$E=\begin{bmatrix}J_1^{-1}J_2 & J_1^{-1}A\\ -I_1 & 0_1\\ 0_2 & -1\end{bmatrix}\in R^{(n+1)\times(n-m+1)} \quad (8)$$

where

$$0_1=\begin{bmatrix}\underbrace{0\ \cdots\ 0}_{n-m}\end{bmatrix}^T\in R^{n-m},$$

$$0_2=\begin{bmatrix}\underbrace{0\ \cdots\ 0}_{n}\end{bmatrix}\in R^{1\times n},$$

$$N=\begin{bmatrix}\bar{N} & 0_2^T\\ 0_2 & 1\end{bmatrix}\in R^{(n+1)\times(n+1)}. \quad (9)$$

Then the $(n+1)$-dimensional vector $N((J_1^{-1}A)^T,0_1^T,-1)^T$ is a solution of the following equation

$$\begin{bmatrix}J^T(q) & A(q)\end{bmatrix}X=0 \quad (10)$$

i.e.,

$$J^T(q)\bar{N}\eta(q)=A(q), \quad (11)$$

where

$$\eta(q)=((J_1^{-1}A)^T,0_1^T)^T\in R^n. \quad (12)$$

It is easy to deduce that E is of full rank, which satisfying

$$\begin{bmatrix}J^T(q) & A(q)\end{bmatrix}E=0 \quad (13)$$

Defining y as

$$y=N^{-1}\begin{bmatrix}q\\ -t\end{bmatrix}=(\xi^T,z^T,-t)^T,$$
$$\xi=(\xi_1,\xi_2,\cdots,\xi_m)^T\in R^m,$$
$$z=(z_1,z_2,\cdots,z_{n-m})^T\in R^{n-m}, \quad (14)$$

then the rheonomous affine kinematic constraints (1) can be expressed as

$$\dot{\xi}=-J_1^{-1}J_2\dot{z}+J_1^{-1}A \quad (15)$$

and one can obtain

$$z=B_1\bar{N}^{-1}q, \quad (16)$$

$$\dot{q}=S(q)\dot{z}+\eta, \quad (17)$$

$$\dot{y}=E\begin{bmatrix}\dot{z}\\ 1\end{bmatrix}+\begin{bmatrix}\eta\\ 1\end{bmatrix}, \quad (18)$$

where $B_1=\begin{bmatrix}0 & I_1\end{bmatrix}\in R^{(n-m)\times n}$. From the definition of N, we can easy deduce that ξ_i,z_j are generalized coordinat-es, that is, each ξ_i,z_j are in the set $\{q_1,q_2,\cdots,q_n\}$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n-m$. The z corresponds to the internal state variable, so that (q,z) is sufficient to describe the constrained motion. The system (3) represents the kinematics of a nonholonomic mechanical system.

Let

$$f_s(t,q,\dot{q}) = j_s^T(q)\dot{q} - a_s,\ \ s = 1,2,\cdots,m \quad (19)$$

then

$$\frac{\partial f}{\partial \dot{q}} = j_s(q),\ \ s = 1,2,\cdots,m \quad (20)$$

When the constraint (1) is imposed on the mechanical system, the virtue displacements of the mechanical system are restricted by [16]

$$j_s^T(q)\delta q = 0,\ \ s = 1,2,\cdots,m \quad (21)$$

By multiplying the Lagrange multiplier λ_s on both sides of (21), then summing (21) from $s = 1$ to m, gives

$$(J(q)\lambda)^T \delta q = 0, \quad (22)$$

where $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)^T$ is m-dimensional vector of the Lagrange multipliers.

The Lagrangian function of the system is

$$L(t,q,\dot{q}) = T(t,q,\dot{q}) - U(t,q), \quad (23)$$

where $T(t,q,\dot{q})$ is the kinetic energy and $U(t,q)$ the potential energy. Define the kinetic energy $T(t,q,\dot{q})$ as

$$T(t,q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}, \quad (24)$$

where $M(q) \in R^{n\times n}$ is symmetric and positive definite.

Considering (23) and using DAlembert-Lagrange principle [16], one can get

$$\left(-\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{\partial L}{\partial q} + \bar{B}(q)u'(t)\right)^T \delta q = 0, \quad (25)$$

where $u'(t) = (u_1'(t), u_2'(t), \cdots, u_l'(t))^T$ is the control, $\bar{B}(q) \in R^{n\times l}$ with $l \geq n-m$, which denotes an input transformation matrix, is a full rank matrix, , vector, δq is the first-order variation in q.

Combining (22) and (25), one obtains

$$\left(-\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{\partial L}{\partial q} + \bar{B}(q)u'(t) + J(q)\lambda\right)^T \delta q = 0, \quad (26)$$

Under (26), the dynamics of the mechanical system can be described by the following differential equations ([17]):

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \bar{B}(q)u'(t) + J(q)\lambda. \quad (27)$$

We will now work out the details for the case of Lagrangian (23),

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{d}{dt}(M(q)\dot{q}) = M(q)\ddot{q} + \frac{dM(q)}{dt}\dot{q}, \quad (28)$$

$$\frac{\partial L}{\partial q} = \frac{1}{2}\frac{\partial}{\partial q}[\dot{q}^T M(q)\dot{q}], \quad (29)$$

Substituting (28) and (28) into (27), yields

$$M(q)\ddot{q} + F(q,\dot{q}) + G(q) = J(q)\lambda + \bar{B}(q)u'(t), \quad (30)$$

where

$$F(q,\dot{q}) = \frac{dM(q)}{t}\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}[\dot{q}^T M(q)\dot{q}]$$

denotes the centrifugal and Coriolis forces and

$$G(t,q) = \frac{\partial U(t,q)}{\partial q}$$

represents the gravitational force.

Differentiating the constraints (17) and (18) with respect to t, it can be readily obtained that

$$\ddot{q} = S\ddot{z} + \dot{S}\dot{z} + \dot{\eta}, \quad (31)$$

$$\ddot{y} = B_4(S\ddot{z} + \dot{S}\dot{z} + \dot{\eta}), \quad (32)$$

where $B_4 = \begin{bmatrix} I \\ 0_2 \end{bmatrix} \in R^{(n+1)\times n}$, $I \in R^{n\times n}$. Now, (30) can be rewritted as

$$\begin{bmatrix} M(q) & 0_2^T \\ 0_2 & 1 \end{bmatrix}\ddot{y} + \begin{bmatrix} F(q,\dot{q}) + G(q) \\ 0 \end{bmatrix} = \begin{bmatrix} J(q) \\ A^T(q) \end{bmatrix}\lambda + \begin{bmatrix} \bar{B}(q)u'(t) \\ -A^T(q)\lambda \end{bmatrix} \quad (33)$$

Left multiplying $B_4^T E^T$ on both sides of (33) and eliminating $\ddot{y}$ by (32), it yields

$$S^T MS\ddot{z} + S^T M\dot{S}\dot{z} + S^T M\dot{\eta} + S^T F + S^T G = S^T \bar{B}u' \quad (34)$$

where $S^T MS$ is a positive definite symmetric inertia matrix, $S^T F$ is the centripetal and coriolis matrix, $S^T G$ is the gravitational friction.

Let $H(q) = \begin{bmatrix} S_1(q) & S_1(q)\eta \\ 0_2 & -1 \end{bmatrix} \in R^{(n-m+1)\times(n+1)}$, it is easy to see that $H(q)$ is of full rank, and

$$H(q)E(q) = I$$

$$\dot{z} = B_1 H(q)\dot{y} = S_1(q)\dot{q} - S_1(q)\eta \quad (35)$$

From (31), (34) and (35), the transformed dynamic model can be solved for q¨ to yield

$$\ddot{q} = W_1(q)\dot{q} + W_2(q,\dot{q}) + S(S^T MS)^{-1} S^T \bar{B}u'(t) \quad (36)$$

where

$$W_1(q) = -S(S^T MS)^{-1} S^T M\dot{S}S_1 + \dot{S}S_1$$

$$W_2(q,\dot{q}) = -S(S^T MS)^{-1}(-S^T M\dot{S}S_1\eta + S^T M\dot{\eta} + S^T F + S^T G) - \dot{S}S_1\eta - \dot{\eta}$$

Define $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$, Then system (18) can be expressed by the following dynamics

$$\dot{x} = \begin{bmatrix} \dot{q} \\ W_1(q)\dot{q} + W_2(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} S(S^T MS)^{-1}\tau(t) \quad (37)$$

where $\tau(t) = S^T \tilde{B}u'(t)$.

Remark 1: This method is also applicable to the nonholonomic control systems with rheonomous affine kinematic constraints:

$$J^T(q)\dot{q} = A(t,q) \quad (38)$$

where $J(q) \in R^{n\times m}$, $A(t,q) \in R^m$, and $J(q), A(t,q)$ as follows:

$$J(q) = (j_1(q), j_2(q), \cdots, j_m(q))^T,$$

$$A(t,q) = (a_1(t,q), a_2(t,q), \cdots, a_m(t,q))^T \neq 0.$$

Remark 2: Noting $(\eta^T(q), -1)^T$ is a solution of the following equation

$$\begin{bmatrix} J^T(q) & A(q) \end{bmatrix} X = 0,$$

it follows $\dot{\eta}(q)$ does not contain $\ddot{q}$.

III. CONCLUSIONS

In this paper, a design scheme of a variable structure relay controller guaranteeing the system global stability and finite time convergence for uncertain nonholonomic systems with affine kinematic constraints has been discussed. The main contribution of this paper is that, a general dynamical state-space model of the mechanical system with affine kinematic constraints is derived.a global variable structure relay control scheme with finite time convergence is advised, which can suppress the nonlinear term tending to lower the convergence rate when the initial state is far away from the origin, and $M(q)$ may be with unknown parameters

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