

# Fuzzy Multi-objective Linear Programming Model for Oilfield Production Distribution

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*Abstract—It is important to program properly during process of oilfield development. Based on the fuzzy understanding about the reservoir and the uncertainty in the actual environment, a kind of fuzzy multi-objective optimization model of production distribution was proposed in this paper, which considers the objectives of production maximization, cost minimization and profit maximization. By using the interactive satisfaction method to find the solution of this model and defining a membership degree to determine the solution. Finally, an example shows that the model can better solve the problem of the production allocation for the oilfield company.*

**Keywords-** Oilfield development programming; formatting; Production distribution; Fuzzy multi-objective linear programming; Membership degree

## I. INTRODUCTION

In the process of oilfield development, especially the middle and later periods of the oilfield development, the oil production decline faster, and measures will be input for increasing or stabilizing the rate of oil recovery. At the same time, the crude oil unit operating costs rapidly rising. So it is necessary to reasonable program for oilfield production, and adopts the optimal combination to realize the profit maximization when completing the crude oil task [1]. Many scholars have studied the oilfield development programming from different angles, such as established the linear programming model of oilfield development [2], the multi-objective linear programming model [3]. The limitations of these models are that the parameters are assumed as the constant, it can't better reflect actual situation of oilfield development. Therefore, Gai [4] introduced the stochastic programming to the oilfield development. The examples show that the stochastic programming can be better in oilfield development compared with the deterministic programming. These stochastic models generally take the form of chance constrained programming model, and use the genetic algorithm to solve them based on the stochastic simulation. But the solution is based on assuming the distribution functions of the random variable are known. When the random variables are much more, the process for solving is quite complicated. Li [5] introduced the fuzzy goal programming, established the fuzzy model and determined the membership function of goal. But he

didn't consider all three goals of oil production, costs and benefits, and the process of the solving is also more complicated. Wang put forward a kind of fuzzy multi-objective optimization problem of aircraft typesetting and give the general solving process [6]. But the solving process is more abstract. Based on the above problems, this paper puts forward a kind of fuzzy multi-objective linear programming model for oilfield production distribution, which considers the goals of oil production, costs and profit, and gives the detailed solving process.

The production distribution model is a kind of multi-objective programming model in which the goal and constraint conditions all contain the fuzzy variables. By utilizing the interaction satisfaction [7], the model can be transformed into the simple and effective fuzzy single objective programming. In the process of solving the model, we can get the solution according to the goals of oil production maximum, cost minimum and profit maximum. Then we can use the formula of membership degree to judge the solution is good or bad.

## II. FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING FOR PRODUCTION DISTRIBUTION

In the oilfield management practice, it is often considered the goals are the maximum oil production, minimum cost and benefits of the best. Let  $c_i$  be the unit operating cost of the  $i$ -th oil production plant, yuan/ton;  $Q_i$  is the distributed production for the  $i$ -th oil production plant, ton;  $T_j$  ( $j=1,2,3,4$ ) is the  $j$ -th taxes, including the added-value tax  $T_1$ , the city construction tax  $T_2$ , the additional tax of education  $T_3$ , the resources tax of oil  $T_4$ , yuan/ton;  $P$  is the price of crude oil, yuan/ton;  $I_i$  is the investment of the  $i$ -th oil production plant, yuan. So the production distribution model has the following characteristics.

### A. Goal Functions

- Cost minimum:  $\min \sum_{i=1}^n c_i Q_i$ ;

- Production maximum:  $\max \sum_{i=1}^n Q_i$  ;
- Profit maximum:  $\max \sum_{i=1}^n (\tilde{P} - \sum_{l=1}^4 T_l - c_i) Q_i$  .

B. Decision variables

$Q_i (i=1,2,\dots,n)$  are the decision variables.

C. Constraint conditions

- Investment constraints: Total investment of all the oil production plants can't exceed the sum of total investment of oilfield company, i.e.  $\sum_{i=1}^n I_i \leq I_b$  , where  $I_i$  is the investment of the  $i$ -th oil production plant,  $I_b$  is the investment of oilfield company;
- Decision variable boundary constraints:  $Q_i^a \leq Q_i \leq Q_i^b$  , where  $Q_i^a$  is the lower limit yields of the  $i$ -th oil production plant,  $Q_i^b$  is the upper limit yields of the  $i$ -th oil production plant.

D. Fuzzy parameters

We think of the goal with fuzzy coefficients. That is to say, the unit operation cost  $c_i$  , the price of crude oil  $P$  , taxes  $T_j$  and total investment of all kinds of oil production plants  $I_b$  are the triangular fuzzy number.

So, the optimization model as following:

$$\min \sum_{i=1}^n \tilde{c}_i Q_i, \max \sum_{i=1}^n Q_i, \max \sum_{i=1}^n (\tilde{P} - \sum_{l=1}^4 \tilde{T}_l - \tilde{c}_i) Q_i$$

$$s.t. \begin{cases} \sum_{i=1}^n I_i \leq \tilde{I}_b \\ Q_i^a \leq Q_i \leq Q_i^b, i=1,2,\dots,n \end{cases} \quad (1)$$

III. MODEL SOLUTION

In this paper, the interactive satisfactory method based on the repeated alternation between the analysts and decision-makers, is used to solve the multi-objective programming model [7]. And then the multi-objective programming can be transformed into a single objective programming. Select one as the standard goal in the three objective functions (such as take the first one as the standard goal), and other objective functions will be provided with a reference function values. So the (1) can be transformed into the following model:

$$\min \sum_{i=1}^n \tilde{c}_i Q_i$$

$$s.t. \begin{cases} \sum_{i=1}^n (\tilde{P} - \sum_{l=1}^4 \tilde{T}_l - \tilde{c}_i) Q_i \geq f_2 \\ \sum_{i=1}^n Q_i \geq f_3 \\ \sum_{i=1}^n I_i \leq \tilde{I}_b \\ Q_i^a \leq Q_i \leq Q_i^b \\ \tilde{c}_i \geq 0 \\ Q_i \geq 0, i=1,2,\dots,n \end{cases} \quad (2)$$

Set the fuzzy number in (2) as the triangular fuzzy number, such as  $\tilde{m}=(m^L, m^*, m^R)$  , where  $m^*$  is the most likely value of  $\tilde{m}$  ,  $m^R$  is the maximum value of  $\tilde{m}$  , and  $m^L$  is the minimum value of  $\tilde{m}$  . Set  $\tilde{m}^0$  as the expected value of  $\tilde{m}$  , which is given by

$$\tilde{m}^0 = E(\tilde{m}) \quad (3)$$

So all the fuzzy numbers can be transformed into the constant number and then substitute them into (2). So we can get the classical programming.

Use the method proposed by S. Dempe to determine the membership function of the solution [10].

A. Firstly simplify the fuzzy model as follows.

$$\min h(x, \lambda) [\alpha] = \lambda C_L^T(\alpha)x + (1-\lambda)C_R^T(\alpha)x \quad (4)$$

The fuzzy number  $\tilde{C}$  is denoted as the triangular fuzzy number-----  $(C_L, C_T, C_R)$  , where

$$C_L(\alpha) = (C_T - C_L)\alpha + C_L$$

$$C_R(\alpha) = (C_T - C_R)\alpha + C_R$$

From (4), we can know that, if  $\bar{x}$  is the solution of (2), we can calculate the membership function. We assume that the Matrix of  $B$  is the basic matrix of the coefficient matrix of  $A$  .

B. Determine the membership function of the solution.

Let

$$Num_i(\lambda) = [C^T(\lambda)]_i - [C_B^T(\lambda)I_B^{-1}]_i \quad (5)$$

$$Den_i(\lambda) = [(C_T^T - C^T(\lambda))B I_B^{-1}]_i - [C_T^T]_i + [C^T(\lambda)]_i \quad (6)$$

When  $\lambda \in ([C_R]_i - [C_T]_i) / ([C_R]_i - [C_L]_i), 1]$  , then

$$z_i^+(\lambda) = Num_i(\lambda) / Den_i(\lambda) \quad (7)$$

When  $\lambda \in [0, ([C_R]_i - [C_T]_i) / ([C_R]_i - [C_L]_i)]$  , then

$$z_i^-(\lambda) = Den_i(\lambda) / Num_i(\lambda) \quad (8)$$

Where  $z^{*+} = \min\{\max\{z_i^+\}\}$ ,  $z^{*-} = \max\{z_i^-\}$ , therefore

$$u(\bar{x}) = \left[ \min_{i \in N} \{z^{*+}\} - \max_{i \in N} \{0, z^{*-}\} \right] / 100 \quad (9)$$

The Steps for solving the fuzzy model:

- Select one goal as the standard goal  $f_1$  in the model (1);
- Introduce the reference values of goal function  $f_i$  ( $i=2,3,\dots$ );
- Use (3) to transform the fuzzy numbers into the constant number and transform the model (1) into the single goal programming;
- Find out the membership degree proposed in this paper. If satisfied, stop, or update the reference function values, return to the second step.

#### IV. EXAMPLE ANALYSES

The predicted values of unit operation cost of 11 oil production plants of shengli oilfield in 2012 are listed in the table 1. The data are got by the five methods of fuzzy prediction, grey prediction, point prediction, numerical prediction and dynamic prediction.

According to the prices over the years, the oil price in 2012 is  $\bar{P}=(5485,5656,5873)$ , yuan/ton. Taxes of one ton includes: 1) Value Added Tax (VAT): 17% of crude oil price; 2) City Construction Tax: 7% of VAT; 3) Additional tax of education: 3% of VAT; 4) Resource tax: 14 Yuan/ton in Shengli Oilfield. The Total investment is  $\bar{I}=(142,150,153)$ , hundred million Yuan. The oil production Predicted value can't be less than 2400 ten thousand ton; the total profit can't be less than 90 billion Yuan. According to these data, we get the optimization model:

$$\min \begin{pmatrix} 44\tilde{3}.15Q_1 + 45\tilde{0}.85Q_2 + 48\tilde{5}.7Q_3 + 47\tilde{9}.14Q_4 + \\ 42\tilde{3}.07Q_5 + 51\tilde{4}.2Q_6 + 50\tilde{4}.64Q_7 + 44\tilde{1}.91Q_8 + \\ 65\tilde{2}.31Q_9 + 39\tilde{4}.40Q_{10} + 33\tilde{0}.50Q_{11} \end{pmatrix}$$

$$\max \begin{pmatrix} [56\tilde{5}6-15\tilde{4}6-44\tilde{3}.15]Q_1 + [56\tilde{5}6-15\tilde{4}6-45\tilde{0}.85]Q_2 + \\ [56\tilde{5}6-15\tilde{4}6-48\tilde{5}.7]Q_3 + [56\tilde{5}6-15\tilde{4}6-47\tilde{9}.14]Q_4 + \\ [56\tilde{5}6-15\tilde{4}6-42\tilde{3}.07]Q_5 + [56\tilde{5}6-15\tilde{4}6-51\tilde{4}.2]Q_6 + \\ [56\tilde{5}6-15\tilde{4}6-50\tilde{4}.64]Q_7 + [56\tilde{5}6-15\tilde{4}6-44\tilde{1}.91]Q_8 + \\ [56\tilde{5}6-15\tilde{4}6-65\tilde{2}.31]Q_9 + [56\tilde{5}6-15\tilde{4}6-39\tilde{4}.40]Q_{10} \\ + [56\tilde{5}6-15\tilde{4}6-33\tilde{0}.50]Q_{11} \end{pmatrix}$$

$$\max \sum_{i=1}^n Q_i$$

$$\begin{cases} 11 \\ \sum_{i=1} I_i \leq 90000000000 \\ 3809500 \leq Q_1 \leq 4036300 \\ 2531400 \leq Q_2 \leq 2573300 \\ 2788200 \leq Q_3 \leq 2972600 \\ 2212300 \leq Q_4 \leq 3037700 \\ s.t. \begin{cases} 2501800 \leq Q_5 \leq 2619500 \\ 1829900 \leq Q_6 \leq 2094000 \\ 1636200 \leq Q_7 \leq 1973600 \\ 1615900 \leq Q_8 \leq 1658600 \\ 978230 \leq Q_9 \leq 1012500 \\ 887000 \leq Q_{10} \leq 1090100 \\ 2058000 \leq Q_{11} \leq 2154200 \end{cases} \end{cases}$$

Get the non-inferior solution:

$$Q_1=4036300, Q_2=2573300, Q_3=2972600, Q_4=3037700, \\ Q_5=2619600, Q_6=1829900, Q_7=1653347, Q_8=1658600, \\ Q_9=978230, Q_{10}=1090100, Q_{11}=2154200$$

Then calculate the membership function of the non-inferior solution. Transform the multi-objective programming into a single objective fuzzy programming and get the coefficient matrix:

$$A=[C \ D \ E \ F],$$

$$C = \begin{bmatrix} -3671.8 & -3664.2 & -3629.3 \\ -1 & -1 & -1 \\ 443.15 & 450.85 & 485.7 \end{bmatrix},$$

$$D = \begin{bmatrix} -3635.9 & -3691.9 \\ -1 & -1 \\ 479.14 & 423.07 \end{bmatrix},$$

$$E = \begin{bmatrix} -3600.8 & -3610.4 \\ -1 & -1 \\ 514.2 & 504.64 \end{bmatrix},$$

$$F = \begin{bmatrix} -3673.1 & -3462.7 & -3720.6 & -3784.5 \\ -1 & -1 & -1 & \\ 441.91 & 652.31 & 394.40 & 330.50 \end{bmatrix}$$

Assume that  $B=\{1,2,3\}$ , then

$$B_A = \begin{bmatrix} -3671.8 & -3664.2 & -3629.3 \\ -1 & -1 & -1 \\ 443.15 & 450.85 & 485.7 \end{bmatrix}$$

Get the values by Using Matlab when  $i=4,5,6,7,8,9,10,11$ . According to the (7-11) in the section two, we have

$$\left\{ \begin{array}{l} Den_4(\lambda)=[-2.91+5.82\lambda]\times 10^9 \\ Den_5(\lambda)=[-2.57+5.14\lambda]\times 10^9 \\ Den_6(\lambda)=[-3.12+6.24\lambda]\times 10^9 \\ Den_7(\lambda)=[-3.06+6.14\lambda]\times 10^9 \\ Den_8(\lambda)=[-2.69+5.38\lambda]\times 10^9 \\ Den_9(\lambda)=[-3.97+7.94\lambda]\times 10^9 \\ Den_{10}(\lambda)=[-2.40+4.97\lambda]\times 10^9 \\ Den_{11}(\lambda)=[-2.00+4.00\lambda]\times 10^9 \\ Num_4(\lambda)=[-1.47+18.39\lambda]\times 10^{10} \\ Num_5(\lambda)=[-1.50+18.68\lambda]\times 10^{10} \\ Num_6(\lambda)=[-1.46+18.22\lambda]\times 10^{10} \\ Num_7(\lambda)=[-1.46+18.27\lambda]\times 10^{10} \\ Num_8(\lambda)=[-1.49+18.59\lambda]\times 10^{10} \\ Num_9(\lambda)=[-1.40+17.52\lambda]\times 10^{10} \\ Num_{10}(\lambda)=[-1.51+18.83\lambda]\times 10^{10} \\ Num_{11}(\lambda)=[-1.53+19.15\lambda]\times 10^{10} \end{array} \right.$$

Here  $z^{*+}=\min\{\max_{\lambda\in[0.5,1]} \{z_4^+\}, \max_{\lambda\in[0.5,1]} \{z_5^+\}, \dots, \max_{\lambda\in[0.5,1]} \{z_{11}^+\}\}=66.8550$ ,  
 $z^{*-}=\max\{z_3^-, z_4^-\}=0$ , So  $u(Q)=\frac{\min_{i\in N}\{z^{*+}\}-\max_{i\in N}\{0, z^{*-}\}}{100}=0.67$ .

The membership degree of the solution is  $u(Q)=0.67$ . Therefore, it can be seen that the solution is more accurate.

### V. CONCLUSIONS

According to the production distribution during the process of oilfield development, this paper introduced a fuzzy multi-objective programming, in which the goals and constraint conditions include fuzzy numbers. Then transform the fuzzy model into a single objective programming by using the

interactive satisfactory method. The process of transforming and solving for the model were described in this paper. Get the solution with the production maximum, cost minimum and profit maximum, and the solution was judged by the membership formula.

### REFERENCES

- [1] Zhao Jinzhou, Liu Zhibin. Optimized Model To Plan production Allocation For Gas Field Development and Its Application [J].Natural gas industry,2004, 24(9): 86~89.
- [2] Jiang Houshun, Liu Ce. An Optimized Model For Oil Development programming / Decision Support System[J].Xinjiang petroleum geology,2004,25(2): 185~187.
- [3] LiuZhibin Zhang Jinliang. Optimal Multi-objective Output Structure Models and Its Application In Oilfield Development Programming System [J].Strategy and management,2004, 13(1): 118~121.
- [4] Gai Yingjie, Chen Yueming, Fan Haijun. A Linear Model with Random Parameters for Measure Programming of Oil-Field [J].Fault block oil and gas fields,2000,7(6):32~34.
- [5] Li Xinshi. Fuzzy Goal Programming And Solution For Oilfield Development[J].Science and technology information,2008:229~230.
- [6] Wu Donghua, Xia Hongshan. Fleet Assignment Problem Study Based On Multi-objective Fuzzy Linear Optimization [J].Computer science, 2012,39(1):234~238.
- [7] Sakawa M, Yano H. An interactive fuzzy satisfying method for multi-objective nonlinear programming [J], 1989, 30(1): 221~238.
- [8] Song Yunna. The Research On The Multi-objective Full Coefficient Fuzzy Programming Problems [D], Master’s degree thesis of Lanzhou university of technology,2007.
- [9] Liu Zhibin, Ren Baosheng. Oil System Simulation and Oilfield Enterprises Optimal Development Decisions [M].Petroleum industry press, 2008.
- [10] S. Dempe, A. Ruziyeva. On the calculation of a membership function for the solution of a fuzzy linear optimization problem [J].Fuzzy Sets and Systems, 2012, 188: 58~67.
- [11] Liu Zhibin, Ding Hui. Optimal model for oil production composition of oilfield development programming and its application [J]. Journal of oil, 2004, 25(1):62~65.
- [12] Hsien-Chung Wu. Using the technique of secularization to solve the multi-objective programming problems with fuzzy coefficients [J], Mathematical and Computer Modeling, 2008, 48: 232~248.
- [13] Stefan Chanas, Dorota Kuchta, Multi-objective programming in optimization of interval objective functions-A generalized approach [J]. European Journal of Operational Research, 1996, 94: 594~598.

**Table 1** The predicted values of unit operation cost (yuan)

Oilfield production plant	Fuzzy Prediction	Grey Prediction	Point Prediction	Numerical Prediction	Dynamic Prediction	Fuzzy Number
Gudao	438.1	458.26	428.04	432.7	451.2	(428.04,443.15,458.26)
Gudong	449.2	458.27	444.71	441.5	460.2	(441.5,450.85,460.2)
Shengli	479	505.95	465.45	480.7	495.6	(465.45,485.7,505.95)
Dongxin	514.5	473.63	443.78	486.5	479.4	(443.78,479.14,514.5)
Hekou	422.9	435.44	416.66	410.7	428.6	(410.7,423.07,435.44)
Binnan	511.2	520.9	506.42	521.7	510.8	(506.42,514.2,520.9)
Xianhe	500.2	519.7	490.38	495.6	517.3	(490.38,504.64,519.7)

Linpan	441.5	440	442.26	440.1	445.7	(440,441.91,445.7)
Chunliang	653.5	674.09	634.17	657.1	642.7	(634.17,652.31,674.09)
Zhuangxi	391.4	403.09	385.49	390.8	401.2	(385.49,394.40,403.09)
Haiyang	330.3	320.57	335.17	329.74	336.7	(320.57,330.50,336.7)