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A Cooperative Stochastic Differential Game of Knowledge Technology Development

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Abstract— **The development of knowledge-based capital constitutes a classic case of market failure which calls for cooperative optimization. However, cooperation cannot be sustainable unless there is guarantee that the agreed-upon optimality principle can be maintained throughout the planning duration. This paper derives subgame consistent cooperative solutions for the development of knowledge technology in a stochastic differential game framework. In particular, subgame consistency ensures that as the game proceeds firms are guided by the same optimality principle and hence they do not possess incentives to deviate from the previously adopted optimal behavior. A "payoff distribution procedure" leading to subgame-consistent solutions is derived and a numerical example is presented. This is the first time that subgame consistent cooperative development of knowledge technology is analyzed.**

Keywords-knowledge technology; stochastic differential games; dynamic cooperation; subgame consistency

I. INTRODUCTION

As markets become increasingly globalized and firms become more multinational, collaborative business research and development (R&D) are likely to yield opportunities to quickly create economies of scale and critical mass, to incorporate new skills and technology, and to facilitate resource sharing. A fundamental premise is that collaborative business R&D is pursued because participating firms can readily gain core skills and technology which would be difficult for them to obtain on their own. This is particularly the case for knowledge technology development. A frequently observed property of knowledge technology is the public goods nature of the technology which is non-exclusive and non-rivalry [1]. Examples of knowledge technology include scientific knowledge, technical know-how, computer software, public information systems, research findings, management information and knowledge-based assets (see examples in [2-7]). Studies on provision of public goods can be found in [8-10].

Cooperation provides the possibility of group optimal solutions to the development of knowledge technology by internalizing the positive externalities of knowledge and reducing the cost of development under joint efforts. However, after a certain time of cooperation, it had been observed that some firms may gain sufficient technological expertise that they would do better by breaking away from the collaboration project. This presents a major source of instability in the collaborative scheme.

To enable a cooperation scheme to be sustainable throughout the agreement period, a stringent condition is needed – that of *subgame consistency*. This condition requires that the optimality principle agreed upon at the outset must remain effective in any subgame starting at a later starting time with a realizable state brought about by prior optimal behaviour. Hence the players do not possess incentives to deviate from the cooperative scheme throughout the cooperative duration. The notion of subgame consistency in stochastic cooperative differential games was originated in [11] in which a generalized theorem for the derivation of an analytically tractable "payoff distribution procedure" (PDP) leading to subgame-consistent solutions has been developed. A series of further developments and applications of cooperative games with subgame consistent solutions can be found in [12-15].

In this paper we present a cooperative stochastic differential game of knowledge technology development and show the derivation of subgame consistent solutions. An analytically tractable "payoff distribution procedure" (PDP) leading to subgame-consistent solutions is derived. The paper is organized as follows. Section 2 provides the analytical framework and the non-cooperative outcome of knowledge technology development. Details of a subgame consistent cooperative scheme are presented in Section 3. A numerical example is given in Section 4. Section 5 concludes the paper.

II. Analytical Framework and Non-cooperative Outcome

Consider the case of an industrial sector in which there are n firms. Each firm has two types of productive capital stocks – an ordinary type of capital with private property rights and a knowledge-based capital which could is accessible by other firms. The ordinary capital includes standard productive capitals like machinery, factory buildings and other forms of tangible assets. The knowledge-based capital include capital assets like scientific knowledge, technical know-how, computer software, public information systems, research findings, management information and knowledge-based assets. Let $K_i(s)$ denote the level of ordinary capital stock and $I_i(s)$ denote the quantity of investment input on ordinary capital by firm i at time s, the stock accumulation dynamics of the ordinary capital of firm i is governed by the stochastic differential equation

$$dK^i(s) = [\ I_i(s) - \delta_i K^i(s)\ ]ds + \sigma_i K^i(s)dz_i(s),$$

$$K^i(0) = K_0^i, \tag{1}$$

for $i \in \{1,2,\cdots,n\} \equiv N$,

where δ_i is the rate of depreciation of the firm i's ordinary capital, $z_i(s)$ is an independent Wiener process and σ_i is a scaling constant.

Let $K^0(s)$ denote the level of the knowledge-based capital stock and $w_i(s)$ denote the quantity of investment input by firm i at time s, the stock accumulation dynamics of the knowledge-based capital is

$$dK^0(s) = \left[\ \sum_{j=1}^{n} b_j w_j(s) - \delta_0 K^0(s)\ \right]ds + \sigma_0 K^0(s)dz_0(s),$$

$$K^0(0) = K_0^0, \tag{2}$$

where δ_0 is the rate of obsolescence of the knowledge-based capital, $z_0(s)$ is again an independent Wiener process, σ_0 is a scaling constant and b_j is a nonnegative constant reflecting the effect of firm i's knowledge-based capital investment on the accumulation of the capital stock.

Moreover, the covariance of $z_i(s)$ and $z_j(s)$ is zero for $i \neq j$, and the covariance of $z_i(s)$ and $z_0(s)$ is zero for all $i \in N$.

The firms use ordinary capital and knowledge-based capital as productive inputs. The effective capital input of firm i is a combination of these two inputs in the form of $K^i + \alpha_i K^0$ where α_i is a non-negative constant reflecting the contribution of the knowledge-based capital on firm i's effective capital stock.

The instantaneous payoff to firm i at time instant s is

$$R_i[K^i(s) + \alpha_i K^0(s)] - c_i[I_i(s)] - c_i^0[w_i(s)],$$

$$i \in \{1,2,\cdots,n\} = N, \tag{3}$$

where $R_i[K^i(s) + \alpha_i K^0(s)]$ is the net revenue of firm i given that its ordinary capital stock is $K^i(s)$ and its knowledge-based capital stock is $K^0(s)$, $c_i[I_i]$ is the cost of investing I_i on its ordinary capital, and $c_i^0[w_i]$ is the cost of investing w_i on the knowledge-based capital. Marginal revenue product of capital is non-negative, that is $R_i^{'}(K^i + \alpha_i K^0) \geq 0$, before a saturation level $\overline{K}$ has been reached. Marginal costs of investment are positive and non-decreasing. Moreover, the payoffs are transferable.

The objective of firm $i \in N$ is to maximize its expected net revenue over the planning horizon T, that is

$$E\left\{ \int_0^T \{R_i[K^i(s) + \alpha_i K^0(s)] - c_i[I_i(s)] - c_i^0[w_i(s)]\}e^{-rs}ds + \{q_i^2[K^i(T)] + q_i^2[K^0(T)] + q_i^3\}e^{-rT} \right\} \tag{4}$$

subject to the stock accumulation dynamics (1)-(2), where r is the discount rate, and $\{q_i^1[K^i(T)] + q_i^2[K^0(T)] + q_i^3\}$ with $q_i^1, q_i^2, q_i^3 \geq 0$ being an amount conditional on the capital stocks that firm i would receive at time T.

For the sake of exposition, we use $K(s)$ to denote the vector $(K^0(s), K^1(s), \cdots, K^n(s))$. Acting for individual interests, the firms are involved in a stochastic differential game. In such a framework, a feedback Nash equilibrium has to be sought. Let $\{\phi_i(s,K) = I_i^*(s) \in I^i$ and $\phi_i^0(s,K) = w_i^*(s) \in \Gamma^i$, for $i \in N$ and $s \in [0,T]\}$ denote a set of feedback strategies that brings about a feedback Nash equilibrium of the game (1)-(2) and (4). Invoking the standard techniques for solving stochastic differential games, a feedback solution to the problem (1)-(2) and (4) can be characterized by the following set of Hamilton-Jacobi-Bellman equations (see [15-16]):

$$-V_t^i(t,K) - \frac{1}{2}\sum_{j=0}^{n} V_{K^j K^j}^i(t,K)(\sigma_j K^j)^2$$

$$= \max_{I_i, w_i} \Bigg\{ \ [R_i\,(K^i + \alpha_i K^0)] - c_i\,(I_i) - c_i^0(w_i)]e^{-rt}$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} V^i_{K^j}(t,K)\ [\ \phi_j\,(t,K) - \delta_j K^j\ ]$$

$$+ V^i_{K^i}(t,K)\ [\ I_i\ - \delta_i\,K^i\ ]$$

$$+ V^i_{K^0}(t,K) \left[\ \sum_{\substack{j=1 \\ j \neq i}}^{n} b_j \phi_j^0(t,K) + w_i\ - \delta_0 K^0\ \right] \Bigg\},$$

$$V^i(T,K) = [q_i^1(K^i) + q_i^2(K^0) + q_i^3]e^{-rT},$$

for $i \in N$. (5)

In particular, $V^i(t,K)$ yields the expected payoff of firm i over the period $[t,T]$ when the ordinary capital stocks and knowledge-base capital stock equal K.

Performing the indicated maximization operator in (5) yields:

$$dc_i\,(I_i\,)e^{-rt} / dI_i\ = V^i_{K^i}\,(t.K) \text{ and} \quad (6)$$

$$dc_i^0(w_i\,)e^{-rt} / dw_i\ = b_i V^i_{K^0}(t.K), \qquad \text{for } i \in N. \quad (7)$$

Firm i will invest in its ordinary capital up to the point where the marginal cost of investments equals the marginal benefits of its ordinary capital to the firm. It will invest in the knowledge-based capital up to the point where the marginal cost of investments equals the product of the marginal benefits of the knowledge-based capital and the effectiveness of investment b_i. A Nash equilibrium non-cooperative outcome of public goods provision by the n firms is characterized by the solution of the system of partial differential equations (5).

III. Subgame Consistent Cooperative Scheme

Now consider the case when the firms agree to cooperate and extract gains from cooperation. In particular, they act cooperatively and agree to distribute the joint payoff among themselves according to an optimality principle. If any firm disagrees and deviates from the cooperation scheme, all firms will revert to the noncooperative framework to counteract the free-rider problem in public goods provision. In particular, free-riding would lead to a lower future payoff due to the loss of cooperative gains. Thus a credible threat is in place. To obtain a sustainable cooperative scheme we first investigate the group optimality and individual rationality issues. Then we derive subgame consistent solutions and the corresponding payoff distribution procedure. The proposed cooperation scheme is depicted in Fig. 1 below.

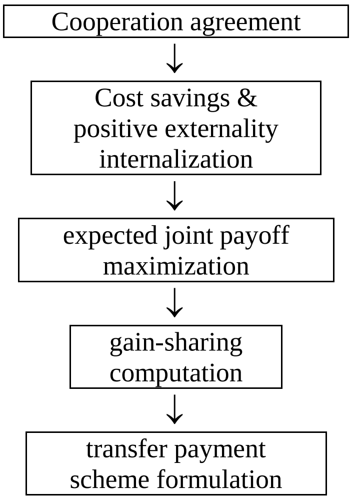

Figure 1. Flow-chart of Proposed Cooperative Plan

A. *Group Optimal Strategies and Individual Rationality*

In cooperative investment in knowledge technology the participating firms can gain core technology knowledge that would be very difficult for them to obtain on their own. Another source of gain from cooperative investment in knowledge technology may come from reduced duplicated effort and economies of scale. These translate into cost savings under cooperation. The joint costs of investment in knowledge technology under cooperation become

$$\hat{c}^0(w_1, w_2, \cdots, w_n) = \sum_{j=1}^{n} \hat{c}_i^0(w_i), \quad (8)$$

where $\hat{c}_j^0(w_j) \leq c_j^0(w_j)$ and

$d\hat{c}_j^0(w_j)/dw_j \leq dc_j^0(w_j)/dw_j$ for all $j \in N$.

To fulfill group optimality the firms would seek to maximize their expected joint payoff. To maximize their expected joint payoff the firms have to solve the stochastic dynamic programming problem of maximizing

$$E\Bigg\{ \sum_{j=1}^{n} \Bigg[\int_0^T \{R_j[K^j(s) + \alpha_j K^0(s)] - c_j[I_j(s)]$$

$$- \hat{c}_j^0(w_j(s)\}e^{-rs} ds$$

$$+ \{q_j^1[K^j(T)] + q_j^2[K^0(T)] + q_j^3\}e^{-rT} \Bigg] \Bigg\} \quad (9)$$

subject to the stock dynamics (1)-(2).

Let $\{\psi_i^0(s,K)$ and $\psi_i\,(s,K)$, for $i \in N$ and $s \in [0,T]\}$ denote a set of strategies that brings about an optimal solution to the stochastic control problem (1)-(2) and (9). Invoking the standard stochastic dynamic programming technique an optimal solution to the stochastic control problem (1)-(2) and

(9) can be characterized by the following set of equations (see [17-18]):

$$-W_t\ (t,K)-\frac{1}{2}\sum_{j=0}^{n} W_{K^jK^j}(t,K)(\sigma_j K^j)^2$$

$$=\max_{\substack{I_1,I_2,\cdots,I_n,\\ w_1,w_2,\cdots,w_n,}}\left\{\ \sum_{j=1}^{n}\left[\ [R_j(K^j+\alpha_j K^0)]\right.\right.$$

$$\left.-c_j(I_j)-\hat{c}_j^0(w_j)\ \right]e^{-rt}$$

$$+\sum_{j=1}^{n} W_{K^j}(t,K)\ [\ w_j-\delta_j K^j\ \ ]$$

$$\left.+W_{K^0}(t,K)\left[\ \sum_{j=1}^{n} b_j w_j-\delta_0 K^0\ \ \right]\ \right\}, \tag{10}$$

$$W(T,K)=\sum_{j=1}^{n}\ [q_j^1(K^j)+q_j^2(K^0)+q_j^3]e^{-rT}. \tag{11}$$

In particular, $W(t,K)$ yields the expected joint payoff of all the firm over the period $[t,T]$ when the ordinary capital stocks and knowledge-base capital stock equal K.

Performing the indicated maximization operator in (10) yields:

$$dc_i\ (I_i\ )e^{-rt}\ /\ dI_i\ =W_{K^i}(t.K)\ \text{and} \tag{12}$$

$$d\hat{c}_i^0(w_i\ )e^{-rt}\ /\ dw_i\ =b_i W_{K^0}(t.K), \qquad \text{for } i\in N. \tag{13}$$

Comparing (7) and (13) shows that investment in knowledge-based capital under cooperation is higher than that under noncooperation because (i) $W_{K^0}(t.K)\geq V^i_{K^0}(t.K)$, that is, the expected marginal gain from knowledge technology by all firms together is higher than that from individual firm i; and (ii) $d\hat{c}_j^0(w_j)/dw_j\leq\ dc_j^0(w_j)/dw_j$, that is lower marginal costs of investment in knowledge technology under cooperation.

A group optimal solution of public goods provision by the n firms is characterized by the solution of the partial differential equation (10)-(11).

Substituting the optimal strategies $\{\psi_i^0(s,K)$ and $\psi_i\ (s,K)$, for $i\in N$ and $s\in[0,T]\}$ into (1) and (2) yields the dynamics of the ordinary capital stocks and that of the knowledge-based capital stock as:

$$dK^j(s)=[\ \psi_j(s,K(s))-\delta_j K^j(s)\ \ ]ds$$

$$+\sigma_j K^j(s)dz_j(s),\ \ K_j(0)=K_j^0,\ \text{for}\ \ j\in N; \tag{14}$$

$$dK^0(s)=\left[\ \sum_{j=1}^{n}\psi_j^0(s,K(s))-\delta_0 K_0^0(s)\ \right]ds$$

$$+\sigma_0 K^0(s)dz_0(s),\quad K^0(0)=K_0^0. \tag{15}$$

We use X_s^* to denote the set of realizable values of $K(s)$ generated by (14) and (15) at time s. The term $K_s^*=(K_s^{0*},K_s^{1*},\cdots,K_s^{n*})\in X_s^*$ is used to denote and element in X_s^*. Moreover, the term K_s^{i*} and $K^{i*}(s)$ are used interchangeably wherever there is no ambiguity.

Let $\xi(\cdot,\cdot)$ denote the agreed-upon imputation vector guiding the distribution of the total cooperative payoff under the agreed-upon optimality principle along the cooperative trajectory $\{K^*(s)\}_{s\in[0,T]}$. At time s and if the productive stock is K_s^*, the imputation vector according to $\xi(\cdot,\cdot)$ is

$$\xi(s,K_s^*)\ =[\xi^1(s,K_s^*),\xi^2(s,K_s^*),\cdots,\xi^n(s,K_s^*)],$$

$$\text{for}\ \ s\in[0,T]. \tag{16}$$

A variety of examples of imputations $\xi(s,K_s^*)$ can be found in [13] and [15]. For individual rationality to be maintained throughout all time $s\in[0,T]$, it is required that each firm's imputed cooperative payoff is no less than its noncooperative payoff, that is

$$\xi^i(s,K_s^*)\ \geq V^i(s,K_s^*),\ \text{for}\ i\in N\ \text{ and }\ s\in[0,T]. \tag{17}$$

To satisfy group optimality, the imputation vector has to satisfy

$$W(s,K_s^*)\ =\sum_{j=1}^{n}\ \xi^i(s,K_s^*),\quad \text{for}\ s\in[0,T], \tag{18}$$

which guarantees that the Pareto optimal joint payoff is shared by the participating firms.

B. Subgame Consistent Solutions and Payoff Distribution Procedure

Under a subgame consistent situation, an extension of the solution policy to a subgame starting at a later time with a state brought about by previous optimal behaviour would remain optimal. For subgame consistency to be satisfied, the imputation $\xi(\cdot,\cdot)$ according to the original agreed-upon optimality principle in (16) has to be maintained along the cooperative trajectory $\{K^*(s)\}_{s\in[0,T]}$.

Following the analysis of [11], [13] and [15], we formulate a Payoff Distribution Procedure so that the agreed-upon imputations (16) can be realized. Let $B^i(s,K^*(s))$ for $s\in[0,T)$ denote the payment that firm i will received at time s under the cooperative agreement if $K^*(s)$ is realized at that time.

The payment scheme involving $B^i(s,K^*(s))$ constitutes a PDP in the sense that along the cooperative trajectory $\{K^*(s)\}_{s\in[0,T]}$ the imputation to firm i covering the duration $[\tau,T]$ can be expressed as:

$$\xi^i(\tau,K_\tau^*) = E\left\{ \int_\tau^T B^i(s,K^*(s))e^{-rs}ds + [q_i^1(K^{i*}(T)) + q_i^2(K^{0*}(T)) + q_i^3]e^{-rT} \middle| K^*(\tau) = K_\tau^* \right\}, \tag{19}$$

for $i \in N$ and $\tau \in [0,T]$.

The values of $B^i(s,K^*(s))$ for $i \in N$ and $s \in [\tau,T)$, which leads to the realization of imputation (16) and hence a subgame consistent cooperative solution can be obtained as follows.

Theorem 3.1. A PDP for firm $i \in N$ with a terminal payment $q_i(K_T^*)$ at time T and an instantaneous payment at time $s \in [0,T]$ which present value is:

$$B_i(s,K_s^*)e^{-rs} = -\xi_s^i(s,K_s^*) - \sum_{j=1}^n \xi_{K^j}^i(s,K_s^*)\left[\sum_{j=1}^n \psi_j(s,K_s^*) - \delta_j K_s^{j*} \right] - \xi_{K^0}^i(s,K_s^*)\left[\sum_{j=1}^n \psi_j^0(s,K_s^*) - \delta_0 K_s^{0*} \right] - \frac{1}{2}\sum_{j=0}^n \xi_{K^jK^j}^i(s,K_s^*)(\sigma_j K_s^{j*})^2, \text{ for } K_s^* \in X_s^*, \tag{20}$$

would lead to the realization of the imputation $\xi(s,K_s^*)$ in (16).

Proof. See Appendix A. ■

Note that the payoff distribution procedure in Theorem 3.1 would give rise to the agreed-upon imputation in (16) and therefore subgame consistency is satisfied.

When all firms are using the cooperative strategies, the payoff that player i will directly receive at time s is

$$R_i(K_s^{i*} + \alpha_i K_s^{0*}) - c_i[\psi_i(s,K_s^*)] - \hat{c}_i^0[\psi_i^0(s,K_s^*)]. \tag{21}$$

However, according to the agreed upon imputation in Theorem 3.1, firm i is supposed to receive $B_i^i(s,K_s^*)$. Therefore a transfer payment (which could be positive or negative)

$$\varpi_i(s,K_s^*) = B_i(s,K_s^*) - \{R_i(K_s^{i*} + \alpha_i K_s^{0*}) - c_i[\psi_i(s,K_s^*)] - \hat{c}_i^0[\psi_i^0(s,K_s^*)]\}. \tag{22}$$

will be imputed to firm $i \in N$ at time $s \in [0,T]$.

IV. A Numerical Illustration

We consider an industry with n asymmetric firms. Each of these firms uses an ordinary capital $K^i(s)$ and a knowledge-based capital $K^0(s)$ to produce its outputs. The accumulation dynamics of these capital stocks are respectively:

$$dK^i(s) = [\, I_j(s) - \delta_i K^i(s) \,]ds + \sigma_i K^i(s)dz_i(s),$$
$$K^i(0) = K_0^i, \tag{23}$$

for $i \in \{1,2,\cdots,n\} \equiv N$, and

$$dK^0(s) = \left[\sum_{j=1}^n b_j w_j(s) - \delta_0 K^0(s) \right]ds + \sigma_0 K^0(s)dz_0(s),$$
$$K^0(0) = K_0^0, \tag{24}$$

Each firm seeks to maximize its expected stream of monetary gains:

$$E\left\{ \int_0^T \{f_i[K^i(s) + \alpha_i K^0(s)] - c_i[I_i(s)]^2 - c_i^0[w_i(s)]^2\}e^{-rs}ds + [q_i^1K^i(T) + q_i^2K^0(T) + q_i^3]e^{-rT} \middle| K(0) = K_0 \right\},$$

for $i \in N$, (25)

subject to (23)-(24);

where f_i, α_i, c_i, c_i^0, q_i^1, q_i^2 and q_i^3 are non-negative constants.

Following the analysis in Section 3 the value function $V^i(t,K)$ can be obtained as:

Proposition 4.1.

The expected non-cooperative payoff of firm i can be obtained as:

$$V^i(t,K) = [A_i(t)K^i + A_i^0(t)K^0 + C_i(t)]e^{-rt} \text{ for } i \in N; \tag{26}$$

where

$$A_i(t) = \left(q_i^1 - \frac{f_i}{r+\delta} \right) e^{-(r+\delta)(T-t)} + \frac{f_i}{r+\delta},$$

$$A_i^0(t) = \left(q_i^2 - \frac{f_i \alpha_i}{r+\delta} \right) e^{-(r+\delta)(T-t)} + \frac{f_i \alpha_i}{r+\delta},$$

and the value of $C_i(t)$ is generated by the following first order linear differential equation:

$$\dot{C}_i(t) = rC_i(t) - \frac{[A_i(t)]^2}{4c_i} + \frac{[A_i^0(t)]^2}{4c_i^0} - \left[\sum_{j=1}^{n} \frac{A_i^0(t) A_j^0(t)}{2c_j} \right],$$

$C_i(T) = q_i^3$, for $i \in N$.

Now we consider the case when the firms agree to act cooperatively and seek higher gains. Cost savings in cooperation yield the cooperative costs of investment in knowledge technology as $\sum_{j=1}^{n} \hat{c}_j^0(w_j)$, where $\hat{c}_j^0$ is a constant which is less than c_j^0. The firms agree to maximize their expected joint gain and distribute the cooperative gain proportional to their non-cooperative gains.

Following the analysis in Section 3, the expected joint payoff of the all the firms $W(t,K)$ can be obtained as:

Proposition 4.2.

$$W(t,K) = [\hat{A}_i(t)K^i + \hat{A}_0(t)K^0 + \hat{C}(t)]e^{-rt}, \qquad (27)$$

where

$$\hat{A}_i(t) = \left(\sum_{j=1}^{n} q_1^j - \frac{\sum_{j=1}^{n} f_j}{r+\delta} \right) e^{-(r+\delta)(T-t)} + \frac{\sum_{j=1}^{n} f_j}{r+\delta},$$

$$\hat{A}_0(t) = \left(\sum_{j=1}^{n} q_j^2 - \frac{\sum_{j=1}^{n} f_j \alpha_j}{r+\delta} \right) e^{-(r+\delta)(T-t)} + \frac{\sum_{j=1}^{n} f_j \alpha_j}{r+\delta},$$

and the value of $C(t)$ is generated by the following first order linear differential equation:

$$\dot{C}(t) = rC(t) - \sum_{j=1}^{n} \frac{[\hat{A}_0(t)]^2}{4\hat{c}_j^0} - \sum_{j=1}^{n} \frac{[\hat{A}_i(t)]^2}{4c_j},$$

$$C(T) = \sum_{j=1}^{n} q_j^3.$$

The accumulation dynamics of the ordinary capital stock and knowledge-based capital stock can be obtained as:

$$dK^i(s) = \left[\frac{\hat{A}_i(s)}{2c_i} I_j(s) - \delta_i K^i(s) \right] ds + \sigma_i K^i(s) dz_i(s),$$

$$K^i(0) = K_0^i, \text{ for } i \in N, \qquad (28)$$

$$dK^0(s) = \left[\sum_{j=1}^{n} b_j \frac{\hat{A}_0(s)}{2c_j^0} - \delta_0 K^0(s) \right] ds + \sigma_0 K^0(s) dz_0(s), \; K^0(0) = K_0^0, \qquad (29)$$

We use X_s^* to denote the set of realizable values of $K^*(s)$ generated by (28)-(29) at time s. The term $K_s^* \in X_s^*$ is used to denote and element in X_s^*.

Since the firms agree to distribute the cooperative gain proportional to their non-cooperative gains, we have

$$\xi^i(s,K_s^*) = \frac{V^i(s,K_s^*)}{\sum_{j=1}^{n} V^j(s,K_s^*)} W(s,K_s^*)$$

$$= \frac{A_i(s)K_s^{i*} + A_i^0(s)K_s^{0*} + C_i(s)}{\sum_{j=1}^{n} [A_j(s)K_s^{j*} + A_j^0(s)K_s^{0*} + C_j(s)]} \times [\hat{A}_i(t)K^i + \hat{A}_0(t)K^0 + \hat{C}(t)]e^{-rt}, \qquad (30)$$

for $i \in N$.

To guarantee dynamical stability in a dynamic cooperation scheme, the solution has to satisfy the property of subgame consistency. Following Theorem 3.1 in Section 3.2 we can obtain the instantaneous payment (in present value) at time $s \in [0,T]$ as:

$$B_i(s,K_s^*)e^{-rs} = -\xi_s^i(s,K_s^*) - \sum_{j=1}^{n} \xi_{K^j}^i(s,K_s^*) \left[\sum_{j=1}^{n} \psi_j^*(s,K_s^*) - \delta_j K_s^{j*} \right] - \xi_{K^0}^i(s,K_s^*) \left[\sum_{j=1}^{n} \psi_j^{0*}(s,K_s^*) - \delta_0 K_s^{0*} \right] - \frac{1}{2} \sum_{j=0}^{n} \xi_{K^j K^j}^i(s,K_s^*)(\sigma_j K_s^{j*})^2, \qquad (31)$$

for $i \in N$ and $K_s^* \in X_s^*$.

The relevant derivatives of $\xi^i(s,K_s^*)$ can be readily obtained using (30).

V. Concluding Remarks

Though cooperative development of knowledge technology captures the gains from positive externalities in productivity and cost savings one may find it hard to be convinced that dynamic cooperation can offer a long-term solution unless the agreed-upon optimality principle can be maintained from the beginning to the end. This paper resolves the classical problem of market failure in the knowledge technology development with a subgame consistent cooperative scheme. The scheme guarantees that the agreed-upon optimality principle can be maintained in any subgame and provides the basis for sustainable cooperation. A "payoff distribution procedure" (PDP) leading to subgame-consistent solutions is developed. A numerical example is presented. This is the first time that subgame consistent cooperative development of knowledge technology is analyzed. Various further research and applications are expected.

Appendix A: Proof of Theorem 3.1.

Invoking (19), one can obtain

$$\xi^i(\tau,K_\tau^*)=E\left\{ \int_\tau^T B_i\,(s,K^*(s))e^{-rs}ds \right.$$

$$\left. +q_i\,[K^*(T)]e^{-rT} \;\middle|\; K^*(\tau)=K_\tau^* \right\},$$

$$=E\left\{ \int_\tau^{\tau+\Delta t} B_i\,(s,K^*(s))e^{-rs}ds \right.$$

$$\left. +\xi^{(\tau+\Delta t)i}(\tau+\Delta t,K_\tau^*+\Delta K_\tau^*) \;\middle|\; K^*(\tau)=K_\tau^* \right\},$$

$i\in N$ and $\tau\in[0,T]$, (32)

where

$\Delta K_\tau^*=\left[\sum_{j=1}^n \psi_j^*(\tau,K_\tau^*)-\delta K_\tau^* \right]\Delta t+\sigma K_\tau^*\Delta z_\tau+o(\Delta t)$, and

$\Delta z_\tau=Z(\tau+\Delta t)-z(\tau)$, and $E_\tau[o(\Delta t)]/\Delta t\to 0$ as $\Delta t\to 0$.

Using (32), one obtains

$$E\left\{ \int_\tau^{\tau+\Delta t} B_i\,(s,K^*(s))e^{-rs}ds \;\middle|\; K^*(\tau)=K_\tau^* \right\}$$

$$=E\left\{ \xi^i(\tau,K_\tau^*) \right.$$

$$\left. -\xi^{(\tau+\Delta t)i}(\tau+\Delta t,K_\tau^*+\Delta K_\tau^*) \;\middle|\; K^*(\tau)=K_\tau^* \right\},$$

for all $\tau\in[0,T]$ and $i\in N$. (33)

If the imputations $\xi^i(\tau,K_\tau^*)$ are continuously differentiable, then as $\Delta t\to 0$, one can express condition (33) as:

$$E\left\{ B_i\,(s,K_s^*)e^{-r\tau}\Delta t+o(\Delta t) \right\}=E\left\{ -\xi_\tau^i(\tau,K_\tau^*)\,\Delta t \right.$$

$$-\xi_{K_\tau}^i(\tau,K_\tau^*)\left[\sum_{j=1}^n \psi_j^*(\tau,K_\tau^*)-\delta K_\tau^* \right]\Delta t$$

$$-\frac{1}{2}\xi_{K_\tau}^i(\tau,K_\tau^*)\,\sigma K_\tau^*\,\Delta z_\tau$$

$$\left. -\frac{1}{2}\xi_{K_\tau K_\tau}^i(\tau,K_\tau^*)\,\sigma^2(K_\tau^*)^2\,\Delta t\;o(\Delta t) \right\},$$

for $i\in N$. (34)

Dividing (34) throughout by Δt, with $\Delta t\to 0$, and taking expectation yield (20). Thus the payoff distribution procedure in $B_i^i(s,K_s^*)$ in (20) would lead to the realization of $\xi(s,K_s^*)$ in (16). ■

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References

[1] P.A. Samuelson, "The pure theory of public expenditure." Rev. Econ. Stat., vol. 36, pp.350-356, 1954.

[2] A. Abecker, A. Bernardi, K. Hinkelmann, O. Kuhn, M. Sintek, "Toward a technology for organizational memories". IEEE intell syst app, vol. 13, pp. 40–48, 1998.

[3] E. Bolisani, and E. Scarso, "Information technology management: a knowledge-based perspective." Technovation, vol. 19, pp.209–217, 1999.

[4] R. Dekker, and R. Hoog, "The monetary value of knowledge assets: a micro approach." Expert Syst. Appl., vol. 18, pp.111–124, 2000.

[5] A.C. Garavelli, M. Gorgoglione, and B. Scozzi, "Managing knowledge transfer by knowledge technologies." Technovation, vol. 22, pp.269–279, 2002.

[6] K.E. Sveiby, The New Organizational Wealth: Managing and Measuring Knowledge-based Assets. Bezett Koekler Publishers, San Francisco, 1997.

[7] C.Y. Wang, H.Y. Yang and S.C.T. Chou, "Using peer-to-peer technology for knowledge sharing in communities of practice." Decis. Support. Syst., vol. 45, no. 3, pp.528–540, 2008.

[8] E. Dockner, S. Jorgensen, N.V. Long, and G. Sorger, Differential games in economics and management science. Cambridge University Press, Cambridge, 2000.

[9] W.K. Wang and C.O. Ewald, "Dynamic voluntary provision of public goods with uncertainty: a stochastic differential game model." Decis. Econ. Fin., vol. 3, pp.97-116, 2010.

[10] D.W.K. Yeung and L.A. Petrosyan, "Subgame Consistent Cooperative Provision of Public Goods." Dynamic Games Appl., forthcoming in 2013.

[11] D.W.K. Yeung and L.A. Petrosyan, "Subgame consistent cooperative solution in stochastic differential games." J. Optim. Theory Appl., vol. 120, no. 3, pp.651-666, 2004.

[12] D.W.K. Yeung, "Dynamically consistent cooperative solution in a differential game of transboundary industrial pollution." J. Optim. Theory Appl., vol. 134, pp.143-160, 2007.

[13] D.W.K. Yeung and L.A. Petrosyan, Cooperative stochastic differential games. Springer-Verlag, New York, 2006.

[14] D.W.K. Yeung and L.A. Petrosyan, "A cooperative stochastic differential game of transboundary industrial pollution." Automatica, vol. 44, no. 6, pp.1532-1544, 2008.

[15] D.W.K. Yeung and L.A. Petrosyan, Subgame consistent economic optimization: an advanced cooperative dynamic game analysis. Boston, Birkhäuser, 2012.

[16] T. Başar, and G.J. Olsder, Dynamic noncooperative game theory. Academic Press, London/New York, 1995.

[17] W.H. Fleming, and R.W. Rishel, Deterministic and stochastic optimal control. Springer-Verlag, New York, Heidelberg and Berlin, 1975.

[18] S. Ross, Introduction to stochastic dynamic programming. Academic Press, New York, 1983.