

The Research and Application of Digital Image Inpainting Based on Improved CDD model

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Abstract—This article studies a new image restoration model, by introducing the edge weighting function and improving the diffusion coefficient of the CDD (curvature-driven diffusions) model, to solve that the CDD model needs long time and has unnatural edge transitions. Empirical results show that the repairing time of the proposed model is less than the traditional model, the repaired image is clearer and the transition is more natural.

Keywords-Digital Image Inpainting, Partial Differential Equation, CDD Model

I. INTRODUCTION

Digital image inpainting initially originated from the Renaissance in Europe, in order to recover some missing or defect parts in artworks, people begun to inpaint them and filled cracks especially for those medieval artworks. In 2000, on the international conference in Singapore, the terminology ‘digital image inpainting’ was firstly put forward by Bertalmio, Sapiro, Caselles and Ballester [1]. The digital image inpainting based on Partial Difference Equation (PDE) is the mainstream image restoration algorithm, and repairs the image by numerical iteration. Foreign typical algorithms based on PDE contain: the BSCB (Bertalmio-Sapiro-Caselles-Ballester) [1] model; the TV (total variation) model [2], which was proposed by Chan and his team; the CDD model [3]; the Euler’s elastica model [4]; the Mumford-Shah model [5]; the Mumford-Shah-Euler model [6]. In domestic, a model based on RBF was proposed by Zhou Tingfang [7]; the p-Laplace inpainting model which was calculated from p-Laplace operator by Zhang Hongying [8-10]; the accelerated texture synthesis algorithm was proposed by Doctor Xu Xiaogang [11,12]; a novel inpainting model based on partial differential equation contains five sub-models was proposed by Jiansheng Liu [17].

An inpainting model based on Curvature Driven Diffusions (CDD) was proposed by Chan and Shen, that has been overcoming the ladder effect generated from TV model. However, it has bigger gradients around the edge, and also has a larger curvature in inpainting field. While the transfer

coefficient of CDD model is $v = g(|k|) |\nabla u|^{-1}$, which means the larger curvature the bigger diffusion, and the greater gradient the smaller diffusion. Therefore, in edge the corner point of inpainting field, the effect for diffusion from curvature and gradient goes to zero. For the straight edge point, curvature and diffusion are basic near to zero, which lead CDD model performs slowly, meanwhile, CDD model has many shortcomings, such images for plenty texture and step edge and big pieces of inpainting field will be obscure and unnatural.

Concluded all above problems about CDD model, we have improved the diffusion coefficient of it and inserted edge weight function, then established a novel inpainting model.

II. THE OVERVIEW OF TV MODEL AND CDD MODEL

A. TV model

We have established a normal formula of variation image inpainting model:

$$J_r(u) = \int_{\Omega} r |\nabla u| dx dy + \frac{\lambda_D}{2} \int_{\Omega} |u - u_0|^2 dx dy \quad (1)$$

Where $\lambda_D = \lambda_D(x) = \begin{cases} 0, x \in D \\ \lambda, x \in \Omega \setminus D \end{cases}$, $u(x, y)$ is the

gray image function, which is shorted for u , u_0 is the defect image grayscale function, D is the area that is in need of restoration, Ω is the entire image area, $\Omega \setminus D$ is the known image field, λ is the Lagrange operator, the equation $d\delta = dx dy$ is a double integral.

In the equation (1), we have set the energy functional of TV model $r[|\nabla u|] = |\nabla u|$ that is:

$$J_r(u) = \int_{\Omega} |\nabla u| \, dx dy + \frac{\lambda_D}{2} \int_{\Omega} |u - u_0|^2 \, dx dy \quad (2)$$

After calculating out the extreme value of formula (2), we have obtained Euler-Lagrange equation of TV model is:

$$-\nabla \cdot \left[\frac{\nabla u}{|\nabla u|} \right] - \lambda(u - u_0) = 0 \quad (3)$$

B. CDD model

In order to make TV model keep a better visual connectivity, take an addition of CDD model, and have modified them, especially diffusion coefficients of TV model, we use $v = g(|k|) |\nabla u|^{-1}$ instead of $v = |\nabla u|^{-1}$, then we can get that

$$g(|k|) = \begin{cases} 0, & k = 0 \\ \infty, & |k| = \infty \\ > 0, & 0 < |k| < \infty \end{cases} \quad (4)$$

Where $k = \nabla \cdot \left[\frac{\nabla u}{|\nabla u|} \right]$ is curvature, generally speaking,

$g(s) = s^a, s > 0, a \geq 1$, we set $a = 1$, because it can make the diffusion be stronger at the large curvature but be weaker at the small curvature, then the corresponding CDD model is:

$$\begin{cases} \frac{\partial u}{\partial t} = \nabla \cdot \left[\frac{g(|k|)\nabla u}{|\nabla u|} \right] + \lambda_D(u_0 - u) \\ g(\cdot), \quad k = \nabla \cdot \left[\frac{\nabla u}{|\nabla u|} \right] \end{cases} \quad (5)$$

And the $g(\cdot)$ is a increasing function.

III. THE INTRODUCTION OF IMPROVED CDD MODEL

A. The introduction of improved CDD model

For long inpainting time and defect edge, we take research based on TV model and CDD model. Firstly, we have separated $|\nabla u|^{-1}$ from the diffusion coefficient, and obtain the conductivity factor $v = g(|k|)$, which can lead velocity of inpainting goes fast, and give out the situation of $|\nabla u|^{-1}$ which may near to 0 and bring singularity which is good for realization and stability of numerical method [13, 14]. Secondly, we introduce the weight function $f(|\nabla u|)$, which

uses gradient mold as parameter to make $f(|\nabla u|)$ to be smaller in the field of huge grads while discrete the smoothness and protect the edge; but in the field of small grads, $f(|\nabla u|)$ is very big which can achieve smoothness and wipe out the noise [15]. So the new model is that:

$$\begin{cases} \frac{\partial u}{\partial t} = f(|\nabla u|) \cdot \nabla \cdot [g(|k|)\nabla u] + \lambda_D(u_0 - u) \\ g(|k|) \\ f(|\nabla u|) \\ k = \nabla \cdot \left[\frac{\nabla u}{|\nabla u|} \right] \end{cases} \quad (6)$$

Where $g(|k|)$ is an increasing function of non-negative and $f(|\nabla u|)$ is a decreasing function of non-negative.

In generally, we choose that $g(|k|) = |k|^a, k > 0, a \geq 1$, $f(|\nabla u|) = [1 + (|\nabla u|/u(i, j))^2]^{-1}$, $u(i, j)$ is the pixel point of gray value of (i, j) , by means of the grads in the field of smooth color of image are small, the grads in the field where gray scale change in severe that can keep sharp-edge and the transition of edge can keep natural.

B. Discretization of Model

We use the half-point difference method to calculate the diffusion format of partial differential function [2, 16]:

$$f(|\nabla u|) \cdot \nabla \cdot [g(|k|)\nabla u] + \lambda_D(u_0 - u) = 0 \quad (7)$$

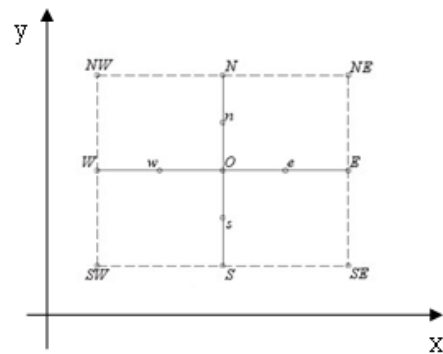


Figure 1. Inpainting pixel points and neighbor fields

In the figure 1, O is the inpainting pixel point, and $\Lambda_0 = \{N, S, W, E\}$ means four neighborhood points set of O , $\Lambda = \{n, s, w, e\}$ means the four half-pixel points. Setting the objective pixel O as $u(i, j)$,

$\Lambda_0 = \{(i, j+1), (i, j-1), (i-1, j), (i+1, j)\}$. When calculating curvature $k = \nabla \cdot [\nabla u / |\nabla u|]$, we should avoid $|\nabla u|$ turning to 0, and take the value of $|\nabla u|_\varepsilon = \sqrt{\varepsilon^2 + |\nabla u|^2}$ or $|\nabla u|_\varepsilon = \varepsilon + |\nabla u|$, if ε is small enough, it will not interrupt the performance of TV inpainting model.

Setting $v = (v^1, v^2) = g(|k|)\nabla u$, to center the difference numerical divergence, then take performance about central difference numerical method for divergence,

$$\nabla \cdot v = \frac{\partial v^1}{\partial x} + \frac{\partial v^2}{\partial y} \approx \frac{v_e^1 - v_w^1}{h} + \frac{v_m^1 - v_s^1}{h}, \text{ the length } h = 1;$$

then we can solve the approach values of $v_e^1, v_w^1, v_s^1, v_m^1$:

$$v_e^1 = g(|k_e|) \left[\frac{\partial u}{\partial x} \right]_e \approx g(k_e) \frac{u_E - u_o}{h} \quad (8)$$

$$v_w^1 = g(|k_w|) \left[\frac{\partial u}{\partial x} \right]_w \approx g(k_w) \frac{u_w - u_o}{h} \quad (9)$$

$$v_n^1 = g(|k_n|) \left[\frac{\partial u}{\partial x} \right]_n \approx g(k_n) \frac{u_N - u_o}{h} \quad (10)$$

$$v_s^1 = g(|k_s|) \left[\frac{\partial u}{\partial x} \right]_s \approx g(k_s) \frac{u_s - u_o}{h} \quad (11)$$

From the above equation, we have to calculate out $g(|k_e|), g(|k_w|), g(|k_n|), g(|k_s|)$, while the function of $g(|k|)$ is ready known, calculating the value of k_q (q is the half-pixel point of Λ , $q \in \{n, s, w, e\}$) and $g(|k_e|), g(|k_w|), g(|k_n|), g(|k_s|)$. With same principle, we can calculate k_q by the use of the difference method.

Because $k = \nabla \cdot [\nabla u / |\nabla u|]$, we can get that

$$\frac{\nabla u_q}{|\nabla u_q|} = \left(\frac{\nabla u_q^1}{|\nabla u_q^1|}, \frac{\nabla u_q^2}{|\nabla u_q^2|} \right) = (v_q^1, v_q^2) \quad (12)$$

Then

$$k_q = \nabla \cdot \left(\frac{\nabla u_q^1}{|\nabla u_q^1|}, \frac{\nabla u_q^2}{|\nabla u_q^2|} \right) = \frac{\partial v_q^1}{\partial x} + \frac{\partial v_q^2}{\partial y} \quad (13)$$

And

$$k_e = \frac{\partial v_e^1}{\partial x} + \frac{\partial v_e^2}{\partial y} = \frac{v_{eE}^1 - v_{eO}^1}{h} + \frac{v_{eNE}^2 - v_{eSE}^2 + v_{eN}^2 - v_{eS}^2}{4h} \quad (14)$$

Apparently, we cannot calculate out v_{eE}^1 in the neighborhood field of 3×3 pixel points (shown as figure 2), but the neighbor field of 5×5 (in figure 3 and 4) can.

NW	N	NE
W	O	E
SW	S	SE

Figure 2. Pixel point O 's 3×3 neighborhood

.	.	O_5	O_4	O_3
.	NW	N	NE	O_2
.	W	O	E	O_1
.	SW	S	SE	O_6
.

Figure 3. Pixel point O 's 5×5 neighborhood

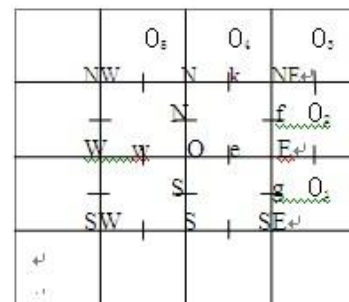


Figure 4. Inpainting pixel points and neighbor fields

And e is the inpainting pixel point, then we can calculate:

$$v_{eE}^1 = \frac{1}{|\nabla u_E|} \left[\frac{\partial u}{\partial x} \right]_E = \frac{1}{|\nabla u_E|} \frac{u_{o_1} - u_o}{2h} \quad (15)$$

$$|\nabla u_E| = \sqrt{\varepsilon^2 + \left(\frac{u_{o_1} - u_o}{2h} \right)^2 + \left(\frac{u_{NE} - u_{SE}}{2h} \right)^2}$$

$$v_{eO}^1 = \frac{1}{|\nabla u_O|} \left[\frac{\partial u}{\partial x} \right]_O = \frac{1}{|\nabla u_O|} \frac{u_E - u_W}{2h} \quad (16)$$

$$|\nabla u_O| = \sqrt{\varepsilon^2 + \left(\frac{u_E - u_W}{2h} \right)^2 + \left(\frac{u_N - u_S}{2h} \right)^2}$$

$$\left\{ \begin{aligned} v_{eNE}^1 &= \frac{1}{|\nabla u_{NE}|} \left[\frac{\partial u}{\partial x} \right]_{NE} = \frac{1}{|\nabla u_{NE}|} \frac{u_{o_e} - u_E}{2h} \\ |\nabla u_{NE}| &= \sqrt{\varepsilon^2 + \left(\frac{u_{o_2} - u_E}{2h}\right)^2 + \left(\frac{u_e - u_E}{2h}\right)^2} \end{aligned} \right. \quad (17)$$

Within the same method, we can calculate the value of $v_{eSE}^2, v_{eN}^2, v_{eS}^2$, turn the formulas (15), (16) and (17) and $v_{eSE}^2, v_{eN}^2, v_{eS}^2$ into the formula (14) to obtain k_e . According to the same method, we can get the value of k_w, k_n, k_s , then the value of $g(|k_e|), g(|k_w|), g(|k_n|), g(|k_s|)$ can be calculated easily. So:

$$\begin{aligned} & f(|\nabla u|) \cdot \nabla \cdot [g(|k|)|\nabla u] \\ &= f(|\nabla u|) \cdot (\nabla \cdot v) \\ &= f(|\nabla u|) \cdot \left(\frac{\partial v^1}{\partial x} + \frac{\partial v^2}{\partial y} \right) \\ &= f(|\nabla u|) \cdot \left(\frac{v_e^1 - v_w^1}{h} + \frac{v_n^2 - v_s^2}{h} \right) \\ &= f(|\nabla u|) \cdot \sum_{Q \in \Lambda_Q, q \in \Lambda} g(|k_q|) [u_Q - u_o] \\ &= \sum_{Q \in \Lambda_Q, q \in \Lambda} f(|\nabla u_q|_\varepsilon) g(|k_q|) [u_Q - u_o] \end{aligned} \quad (18)$$

Where $Q \in \Lambda_Q, q \in \Lambda$ illustrate the situation that q is the half-pixel point of Λ , if Q is equal to E , the q will be equal to e . So we can simplify the formula (7) to be:

$$\begin{aligned} & \sum_{Q \in \Lambda_Q, q \in \Lambda} f(|\nabla u_q|_\varepsilon) g(|k_q|) [u_Q - u(i, j)] \\ &+ \lambda_D(i, j)(u_0(i, j) - u(i, j)) = 0 \end{aligned} \quad (19)$$

Rationally,

$$\begin{aligned} u(i, j) &= \frac{\sum_{Q \in \Lambda_Q, q \in \Lambda} f(|\nabla u_q|_\varepsilon) g(|k_q|)}{\sum_{Q \in \Lambda_Q, q \in \Lambda} f(|\nabla u_q|_\varepsilon) g(|k_q|) + \lambda_D(i, j)} u_Q \\ &+ \frac{\lambda_D(i, j)}{\sum_{Q \in \Lambda_Q, q \in \Lambda} f(|\nabla u_q|_\varepsilon) g(|k_q|) + \lambda_D(i, j)} u_0(i, j) \end{aligned} \quad (20)$$

Where

$$\omega_q = f(|\nabla u_q|_\varepsilon) g(|k_q|) \quad (21)$$

$$h_Q = \frac{\omega_q}{\lambda_D(O) + \sum_{Q \in \Lambda_Q, q \in \Lambda} \omega_q} \quad (22)$$

$$h_o = \frac{\lambda_D(O)}{\lambda_D(O) + \sum_{Q \in \Lambda_Q, q \in \Lambda} \omega_q} \quad (23)$$

So the formula (20) can be remarked as:

$$u(i, j) = \sum_{Q \in \Lambda_Q, q \in \Lambda} h_Q u_Q + h_o u_o(i, j) \quad (24)$$

While $\sum_{Q \in \Lambda_Q, q \in \Lambda} h_Q + h_o = 1$; apparently, the inpainting objective pixel value $u(i, j)$ add weight h_Q to be repaired through neighbor field point u_Q . We have used Gauss-Jacobi iteration method, and the formula (24) can be shown as:

$$u^{(n)}(i, j) = \sum_{Q \in \Lambda_Q, q \in \Lambda} h_Q^{(n-1)} u_Q^{(n-1)} + h_o^{(n-1)} u_o(i, j) \quad (25)$$

Where n is the number of iterations, and $\sum_{Q \in \Lambda_Q, q \in \Lambda} h_Q + h_o = 1$.

The process of this inpainting model mainly has shown as following:

Initialized $u^0(i, j)$, generally we set $u^0(i, j) = u_o(i, j)$;

For $n=1, 2, \dots$

For $i=1, 2, \dots, M$

For $j=1, 2, \dots, N$

By the use of formula (21), (22) and (23), calculate ω_p, h_p, h_o respectively, the (i, j) as the center of them, then use formula (25) to calculate.

$$u^{(n)}(i, j) = \sum_{Q \in \Lambda_Q, q \in \Lambda} h_Q^{(n-1)} u_Q^{(n-1)} + h_o^{(n-1)} u_o(i, j)$$

End

End

End

C. Model simulation

On the level of Matlab 7.1, we have performed experiments among improved CDD model and TV model and original CDD model. Firstly, we have set mask code in the inpainting field D ; Secondly, we have inpainted the image of inpainting

model based on the known information in the inpainting field. Setting $\lambda = 0$, $\varepsilon = 0.0001$ with improved signal-to-noise (ISNR) as the objective metric standard of the quantity of inpainted image, the definition of ISNR as:

$$ISNR = 10 \cdot \lg \left\{ \frac{\sum_{x=1}^M \sum_{y=1}^N [\hat{I}(x, y) - I^0(x, y)]^2}{\sum_{x=1}^M \sum_{y=1}^N [\hat{I}(x, y) - I(x, y)]^2} \right\}$$

Where $\hat{I}(x, y)$ is the original gray image, $I^0(x, y)$ is the damaged image, $I(x, y)$ is the recovered image.

1) Visual connectivity

Based on relatively theory, TV model not only can generate ladder effect easily, but also bring bad visual connectivity. However, CDD model can overcome them. Then this paper study the improved inpainting method mainly based on CDD model. In the following figure, we have experiments on each model to inpaint the original image in Figure 5(a), while the iteration number in $n = 100$.

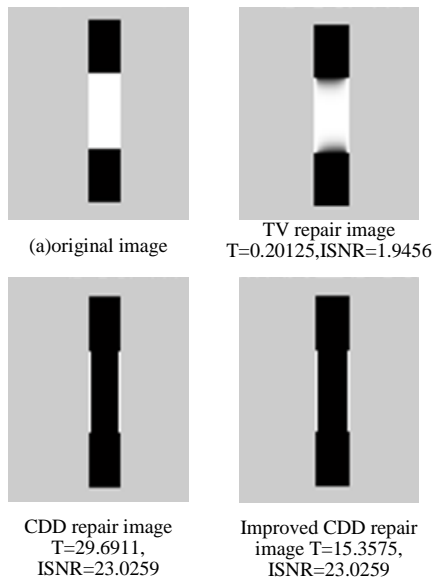


Figure 5. Comparing effect figure of each inpainting model about visual connectivity

The experiments show that that TV model can't content connectivity, but CDD model and the improved CDD model can content it, during the same iteration number and inpainting effect, the inpainting time of improved CDD model goes faster than the CDD with 0.5 times. The following table 1(A), (B) are the respectively relationship table of the inpainting image of each inpainting model in Figure 5(a) with the iteration number N and inpainting time T and ISNR. After analyzing the table, the effect of Figure 5(a) have no effect on iteration number, the bigger the iteration number and the longer the inpainting time,

there is no change on the inpainting effect (ISNR=23.0259), during the inpainting process of Figure 5(a), the value of improved CDD model and the original CDD model are the same.

TABLE 1(A) N-ISNR TABLE

The ISNR comparison of the three models			
	TV	CDD	Improved CDD
N=20	ISNR=0.72857	ISNR=23.0259	ISNR=23.0259
N=40	ISNR=1.1796	ISNR=23.0259	ISNR=23.0259
N=80	ISNR=1.7407	ISNR=23.0259	ISNR=23.0259
N=100	ISNR=1.9456	ISNR=23.0259	ISNR=23.0259

TABLE 1(B) N-T TABLE

The T comparison of the three models			
	TV	CDD	Improved CDD
N=20	T=0.062094	T=5.7626	T=3.019
N=40	T=0.11557	T=11.4937	T=6.0553
N=80	T=0.23172	T=23.1025	T=12.1578
N=100	T=0.28324	T=28.7996	T=15.3199

2) Scratch noise inpainting

a) Roughly scratching inpainting

Setting iteration number n=100, the inpainting effects of rough scratches in each model are shown in Figure 6.

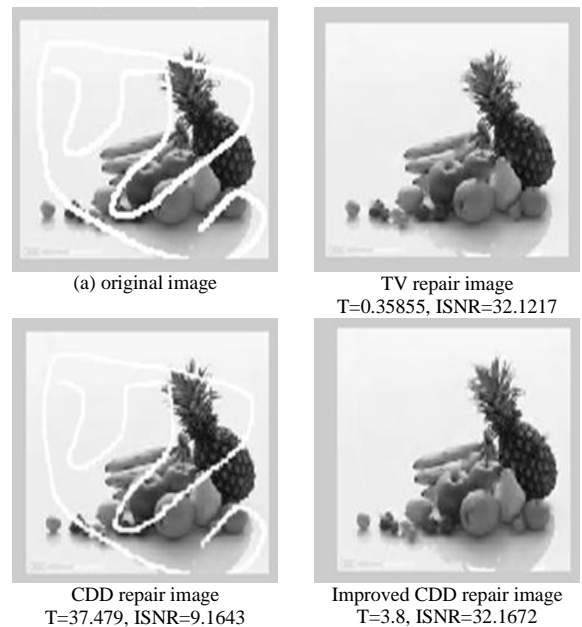


Figure 6. Comparing effect figure of each inpainting model for denoising

Table 2(A) and (B) are the relationship table when iteration number is N and inpainting time T, ISNR are the same to Figure 6(A). According to table 2, we obtain six times of iteration data result, drawing ISNR-N relationship figure and

T-N relationship figure respectively and they are shown in Figure 7.

TABLE 2(A) N-ISNR TABLE

The ISNR comparison of the three models			
	TV	CDD	Improved CDD
N=20	ISNR=30.3422	ISNR=4.346	ISNR=16.0275
N=40	ISNR=32.0544	ISNR=5.8635	ISNR=26.3596
N=80	ISNR=32.1231	ISNR=8.0544	ISNR=31.8596
N=100	ISNR=32.1217	ISNR=9.1643	ISNR=32.1672

From table 2, after six times of iteration with Matlab7.1, we have drawn the N-ISNR relationship figure and N-T relationship figure respectively in Figure 7. Inner the figure, using * connect the curve is TV model, using \diamond connect the curve is improved CDD mode, and \circ means CDD model, the followings are the same principle.

TABLE 2(B) N-T TABLE

The T comparison of the three models			
	TV	CDD	Improved CDD
N=20	T=0.13875	T=7.8845	T=0.76341
N=40	T=0.21323	T=15.8081	T=1.978
N=80	T=0.43322	T=31.8532	T=3.9815
N=100	T=0.53377	T=39.4456	T=3.8578

Analyzing the table 2 and Figure 7, for TV model, the inpainting time seems stable relatively, but the effect of changes little, when the iteration number is more than 80, the ISNR stay in some degree stable; for the original CDD model, the inpainting time and its ISNR increase with N increased, but the effect of the inpainting changes fast, however, with the same N, the inpainting time is longer than TV model and improved model, and the effect changes little; for the improved CDD model, when the iteration number goes bigger than 80, the inpainting effect changes to stable and better than TV model, the inpainting time looks more stable respectively, there is little change on it.

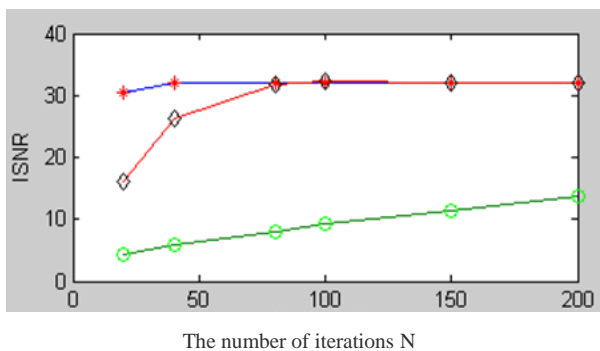


Figure 7(a). N-ISNR relationship figure

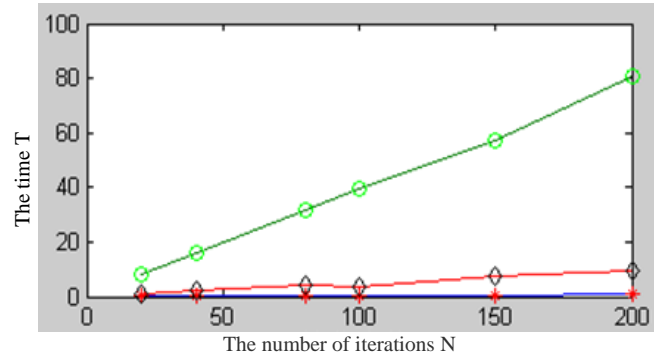


Figure 7(b). N-T relationship figure

b) Small scratch inpainting

Setting iteration number $n=100$, the figure for the inpainting effect of small scratch in each model is shown in the figure 8. Table 3(A) and (B) are the relationship table of Figure 8(A) when the iteration number is N and inpainting time is T and ISNR, meanwhile, according to table 3, setting six iteration data result to draw out ISNR-N relationship figure, which is shown in the figure 9.

Analyzing the above table 3 and figure 9, the performance of TV model is best apparently, because its inpainting time is shortest in three model, and the effect is best; the improved CDD model is better than original CDD model, because its inpainting time and the clarity is better than original one. The inpainting effect of CDD changes fast, while TV model changes slowly when N increased. Above all, the algorithm of improved CDD model can use less time and less iteration number to reach the result and content the visual connection are better than original CDD.

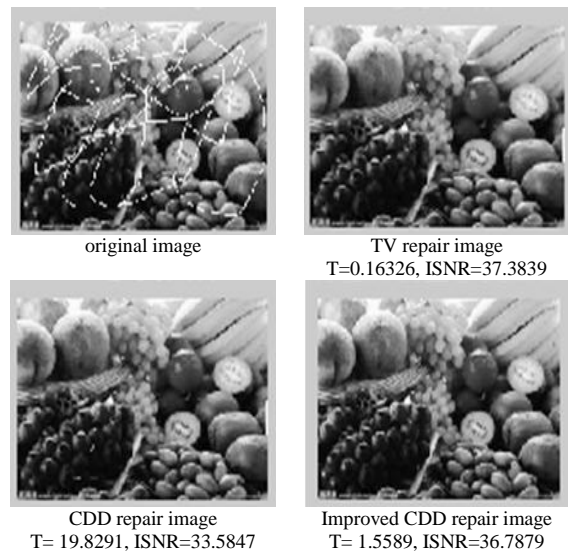


Figure 9(b). N-T relationship figure

c) *Big block field inpainting*

Within the same principle in figure 10, the iteration number is $n=100$, providing there are big block fields in the image, which is the inpainting effect of each model. Table 4(a) and table 4(b) is the relationship table which shows the iteration number N and inpainting time and ISNR. According to the table 4, the iteration number is six, the data results that draw the relationship figure for ISNR- N and T - N is shown in the following.

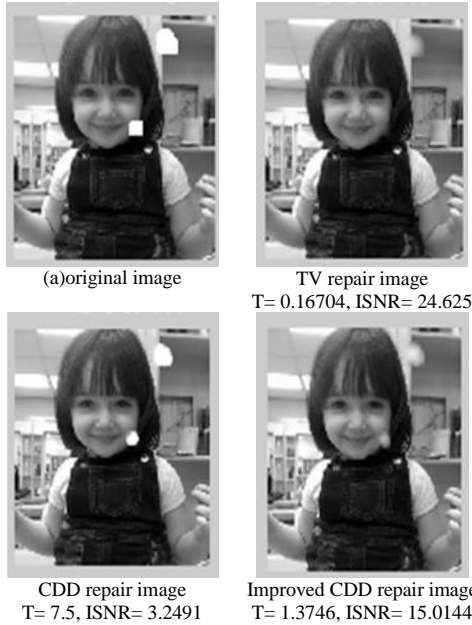


Figure 10. Comparing effect figure of each inpainting model for big block field

TABLE 4(A) N-ISNR TABLE

The ISNR comparison of the three models			
	TV	CDD	Improved CDD
N=20	ISNR=8.4914	ISNR=80828	ISNR=4.8697
N=40	ISNR=14.1429	ISNR=1.68	ISNR=8.1204
N=80	ISNR=21.8556	ISNR=2.7952	ISNR=12.7717
N=100	ISNR=24.625	ISNR=3.2491	ISNR=15.0144

TABLE 4(B) N-T TABLE

The T comparison of the three models			
	TV	CDD	Improved CDD
N=20	T=0.036022	T=1.5407	T=0.28762
N=40	T=0.069329	T=3.1889	T=0.55706
N=80	T=0.13239	T=6.2862	T=1.13178
N=100	T=0.15986	T=7.944	T=1.6135

Analyzing Figure 11, the inpainting figure of ISNR- N and N - T by big block field is different with the above method,

especially about the changing trends. In this case, the ISNR and the time T are going with the iteration N , and the changes of inpainting are apparently. For big block inpainting field images, the larger iteration number, the better the inpainting effect, if the N is set some certain value, ISNR will tend to be stable.

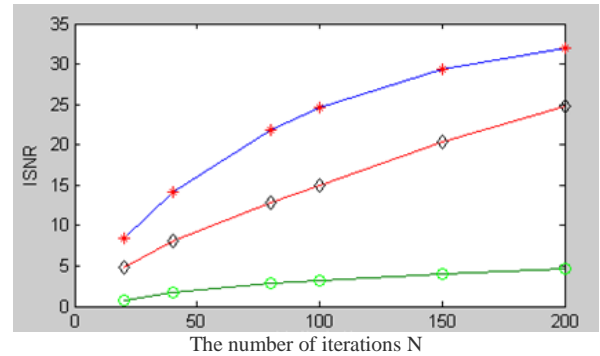


Figure 11(a) N-ISNR relationship figure

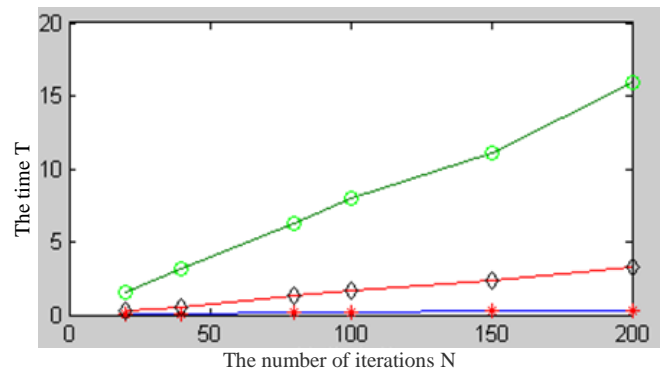


Figure 11(b) N-T relationship figure

D. *Conclusion*

In conclusion, although TV model has the best inpainting effect and the least inpainting time, it can't content the connection; the improved CDD model has the same advantage with TV model, but it can make image be more smoothly on the edge and the connection is very well; however, the original CDD model can content the connection, it needs longer time to repair the image and has bad effect on the image.

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