A Formal Mathematical Model of *Cognitive* Radio

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Abstract— **Cognitive Radio (CR) has caught a lot of attention lately due to its unprecedented capabilities, and its potential to enhance the user's radio experience. CR, as introduced by Mitola, adds a new dimension in the radio domain; that's the incorporation of the capabilities of knowledge representation and processing. Combined with reasoning functionalities the radio is capable of providing a vast range of applications and services; like efficient radio resource management and enhanced environment-aware services. However, in order to characterize the internal functions of a CR and the ultimate possibilities of its applications and services; a formalization of the cognition concept is needed. This formalization enables scientists to carry out rigorous scientific analysis for various CR-related problems. Inspired from cognitive sciences, cognitive neurosciences, and artificial intelligence realms, this paper aims at developing a formal model for CR. For the first time, to our knowledge, formalization for CR has been proposed, thus laying down a preliminary effort towards developing a formal rigorous treatment for all future CR aspects. The resulting formal model is shown to accommodate the CR definitions and concepts developed by the pioneers in this field.**

Artificial Intelligence; Cognition Cycle; Cognitive Radio; Cognitive Sciences; Formal Model; Turing Machine.

I. INTRODUCTION

Cognitive Radio (CR) represents a new paradigm in the history of wireless communications. Every new generation of wireless communication systems usually introduced either new services for the end user, or improved services already provided by older generations [1]. This improvement is in the form of better mobility, greater capacities, and better QoS metrics. However, CR could hold the potential of providing more capabilities beyond the current top notch generations of wireless communications systems; like 4G Standards.

Mitola's implementation for CR incorporated knowledge inside the radio with the ability to employ model based reasoning; thus by which a certain level of competence in radio-related domains could be achieved [2]. This enabled the radio to be used in a wide range of new innovative applications like efficient radio resource management and enhanced environment-aware services [3].

In order to push the degree of possibilities of CR applications and services to the maximum, in addition to develop this technology towards maturity; some form of a rigorous theoretical formalization is needed. A science, or any knowledge field, only comes to maturity after formalization. This formalization enables scientists to carry out rigorous scientific analysis for various CR-related problems. In addition, formalization allows us to better understand the deep aspects and implications of that field. Once formalized, an evolution of the concept using mathematical operators could eventually lead to new concepts and ideas.

This paper aims at developing a formal model for CR inspired from cognitive sciences, cognitive neurosciences, and artificial intelligence realms. Most cognitive sciences are concerned with theories characterizing the functions made possible by the human brain; and usually emphasis on cognition theories is done using biological evidence, inferred from cognitive neurosciences [5]. In our work, we will make use of this methodology in order to lay down our formal theoretical model for CR. We will also build upon expertise gained from previous years of research in artificial intelligence. We believe that it is very much related to Cognitive Machines; and hence to Cognitive Radios.

The paper is organized as follows: Section I covers the introduction to the topic. Section II covers the modularity of the mind and the nature of cognition as defined from the cognitive sciences context. In addition, it illustrates also the biological evidence of the modularity of the mind, and other properties of cognition, as inferred from cognitive neurosciences. In Section III, we provide the proposed inner CR modules through a novel cognition cycle based on findings from cognitive sciences. Section IV exhibits the main formal tools that would assist in modeling the cognitive process. Section V illustrates the formalization efforts done in this work using a novel modified form of a widely used computational model. Section VI demonstrates the capability of the proposed CR theoretical model to accommodate the CR definitions and concepts previously introduced by Mitola and other scientists in the field. Finally, section VII provides the conclusion and future work.

II. NATURE OF COGNITION AND MODULARITY OF THE MIND

Covering the nature of cognition in such a limited space is a very challenging task. However, this task is seen as crucial for laying down a rigorous theoretical model for CR. We begin by posing some questions for which we seek an answer. First, does CR, as currently defined by the scientific literature, really perform the cognitive faculties of the human mind? Did the CR concept, when first introduced by Mitola, include all aspects of cognition, or only a subset of the concept? Does cognition implies or implicitly includes intelligence?

A simple straight forward answer to the above inquiries: A CR must possess the real elements of *cognition* if one wishes to have a real cognitive machine. This should be manifested from its behavior – cognitive behavior – in different situations and contexts. Cognitive science embraces other scientific disciplines like psychology, artificial intelligence, neuroscience, linguistics, and anthropology. Its intellectual origins are in the mid-1950s when researchers in several fields began to develop theories of the mind, based on complex representations and computational procedures [5].

Cognition is generally understood as the scientific study of knowledge, and the way it is acquired, retained, and used as a basis for action [6]. Cognition deals with the way humans recognize objects in their surrounding environment in addition with the capability of filtering out some stimuli while paying attention to others. Cognition also deals with how problems are solved, how conclusions are drawn, and how decisions are made [6,7].

This work aims at illustrating the true meaning of cognition theoretically, and to indentify its elements; i.e. its modules if any, and states. This would assist in the characterization of the main building blocks of CR. The following sub-sections demonstrate that mental faculties, like cognition, employ the simultaneous consideration of many pieces of information or constraints, through some sort of parallel distributed processing [8]. In addition, some cognitive modules can be modeled as a complex structure. They are subject to a process of evolution, through a process of self-organization based on distributed variation and selection process. Hence, central to our flow of reasoning for modeling CR, are the concepts of *modularity*, *parallel distributed processing*, and *evolution*. A brief account is given for each:

A. Modularity

In engineering the idea of modular design is axiomatic [9]. Complex systems are made up of specialized subcomponents designed for particular functions.

The author in [10] gave a very good account on the presence of modularity in mental, biological, and neural structures. For example, references [11-13] shows sufficient evidences that many biological structures are modular, at the physiological and genetic levels. Mental and neural structures, too, shows modularity in structure [14-16], and such modularity might be an essential or highly probable aspect of the evolution of complex systems, at both cognitive [17,18] and neural levels [19-21].

Consistent with this idea, the brain is actually composed of many functional blocks separated from each other; yet connected via strands analogous to fiber optic strands. This ‘internal highway system’ of the cortex – fiber tracts running through the inner brain, is analyzed using a method called *Diffusion Tractography*. This highway enables the functional units to operate completely in parallel, thus enabling parallel processing and very high speed manipulation of sensory data and inter-functional block communications [22,23].

Hence, the concept of modularity is central in cognitive sciences. It is even considered as a basic principle – the principle of modular design –, which accordingly enables us to split cognition into a collection of specialized modules, interacting together for achieving some purpose [16]. Moreover, every module in itself could be composed of sub-modules, according to the module complexity.

Formally, the above property should be modeled by employing a formal model that supports modularity as we will see in the next sections.

B. Parallel Distributed Processing

We have illustrated that cognition consists of a collection of specialized modules each is responsible of performing a certain aspect of cognition. The temporal examination of the human cognitive process reveals a distinct sequential nature. Though the process may not be discrete, it has a distinctly sequential character, with transitions from *state*-to-*state* occurring, say, two or three times a second [8].

Our central question here: ***Is the human cognitive process better modeled by sequential models?***

Every cognition state, itself consists of a number of sub-states, which could be modeled in a number of microsteps. This conforms to the modularity concept discussed in the previous sub-section. Even the simplest macrosteps of cognition—say, recognition of single words—require vast numbers of microsteps if they are implemented sequentially. However, according to Feldman and Ballard [8]:

"the biological hardware is just too sluggish for sequential models of the microstructure to provide a plausible account, at least of the microstructure of human thought [24]."

This means, that the cognitive tasks seem to require parallel distributed processing models. These models are capable of modeling each aspect of the information in the situation while acting on and influencing other aspects simultaneously; and vice versa. Again, evidence from cognitive neurosciences, which supports the above claims, indicates that the brain consists of a large number of highly interconnected blocks which send to each other simple excitatory and inhibitory messages and update their excitations on the basis of these simple messages [8].

Thus formally, the distributed parallel nature of the cognitive process should be modeled by employing a formal model that supports parallel distributed processing.

C. Evolutionary Nature of Cognition

The emergence and evolution of complex structure is seen as the variation of relatively stable system components (modules), through processes like recombination and mutation, combined with the selective retention of stable invariant assemblies. This leads to the formation of higher-order stable systems. This phenomenon is referred to as self-organization [25]. From an evolutionary perspective, stable structures emerging through self-organization could be characterized by a specific combination of modules that have *Closure* properties. A system or a module is closed if all the transformations or relations arising from its internal organization are such that they map the distinction defining the identity of the system upon itself.

Cognition is a process of internal self-organization whose function is to allow an actor to adapt to a complex environment by choosing appropriate action complexes. An autonomous system should be able to reconstruct the stable distinctions relevant for its survival out of the stimuli it receives. The cognitive system "closes" the pattern by filling in the missing elements [25].

The evolutionary nature of the cognitive process should be modeled by employing a formal model that supports evolutionary behavior.

D. Subsumption Architecture

There is another complementing view of the mind, of the brain operation and of intelligence. Rodney Brooks suggested the subsumption architecture [26]. He argued that intelligent behavior could be achieved using a large number of loosely coupled processors that function predominately in an asynchronous, parallel way [27]. Minimal internal processing is required in this case. Sensory signals should be mapped relatively directly to motor signals. This architecture leads to a tight system-environment coupling. Intelligence in this sense arises from the interaction of an organism with its environment. However, central to this idea, is that intelligence should have a body, an agent in this context, which should be autonomous.

Subsumption architecture is a methodology that would facilitate the design of cognitive entities that pursue multiple goals and respond to multiple sensors, that perform robustly, and that are, most importantly, incrementally extendable.

It is evident, that some functions or modules of the mind rely on the subsumption architecture devised by Brooks. Our formal model should take that into consideration in situations where a tight system-environment coupling is envisaged in favor of going through the normal stimulus processing flow.

III. NOVEL COGNITION CYCLE

From the previous discussion, cognitive activities are reflected in three major themes; namely the central role of knowledge in interpreting the environment, the processes by which knowledge gets translated into action, and the principles underlying the learning of facts and acts, strategies and procedures for action [28].

Figure 1 illustrates a state diagram representing the different states of the Cognition Cycle of a CR; where every state in the cycle can be composed of multiple modules, and each state by itself is subject to evolution [29]. This cycle is used by the authors to develop a novel CR adaptation engine architecture inspired from theories developed in cognitive sciences [29].

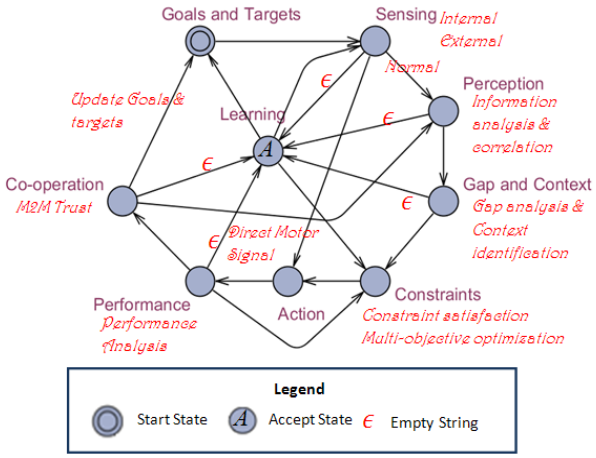

Fig. 1. Novel Cognition Cycle for CR [29]

A state is defined as an internal combination of the values of some internal variables and registers inside one or modules of the machine. The cognition process inside the radio starts by autonomously setting up radio goals which targets – *Goals and Targets* – the high availability, and robustness of the communication process. Next is the sensing stage – *Sensing* – which includes all the sensing operations related to events triggered from the environment or from inside the radio itself. The availability of a huge set of raw data necessitates the existence of a module which extracts – *Perception & Attention* – useful information for further processing. After processing the sensory inputs and identifying the radio goals and targets; gap identification followed by a loop of controlled optimization through a feedback mechanism is performed by the *Gap and Context*, *Constraints*, *Action*, and *Performance* stages respectively.

Central to the cognition cycle is the state of *Learning*; where learning algorithms are implemented in the radio in order to add a learning capability to respond to new unprecedented events facing the radio during its operation. Finally, an important aspect in the concept of cognition is the ability to commune with other nodes through the concept of cognitive networks by means of a *Co-operation* state where mutual cooperation between the CR and other CRs, is achieved to develop collaborative decision-making capabilities.

As indicated in [29], every state having more than one exit path to the other states needs a decision module. Decision must be incorporated with some sort of a reasoning engine for proper choices to be made. This necessitates the use of an intelligent agent [30]. i.e. CR implicitly includes intelligence for proper cognitive behavior.

IV. THEORETICAL COMPUTATIONAL MODELING

Computational models offers a powerful means to simulate the operation of machines. The cognitive process as illustrated by the cognition cycle can ultimately be modeled by a theoretical computational model [31].

Several models of computations are in existence in the literature, each with certain powers and limitations. Of the simplest forms of computational models, are the *Finite State Automata (FSA)*, and of the most generic and powerful models are the *Universal Turing Machines (UTM)* and *Multitape Turing Machine*. Based on a slight modification of the Universal Turing Machine computational model, we will demonstrate in the next section a new computational model – the Universal Multitape Turing Machine – which is deemed more suitable for modeling the cognitive process.

A. Turing Machine(TM)

Formally, a Turing machine (TM) is a 7-tuple, $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where Q, Σ, Γ are finite sets and

1. *Q is the set of states,*
2. *Σ is the input alphabet not containing the blank symbol $\sqcup$,*
3. *Γ is the tape alphabet, where $\sqcup \in \Gamma$, and $\Sigma \subseteq \Gamma$,*
4. *$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is the transition function,*
5. *$q_0 \in Q$, is the start state,*
6. *$q_{accept} \in Q$, is the accept state, and*
7. *$q_{reject} \in Q$, is the reject state*, where $q_{accept} \neq q_{reject}$

The set of head movements $\{L, R\}$ represent the head moving one step to the Left or one step to the Right respectively.

This computational model is a powerful generalization of a FSA; however, it doesn't have the ability to simulate the operation of other machines. This drawback has been resolved by the Universal Turing Machine computational models.

B. Multitape Turing Machine (MTM)

A Multitape Turing Machine is an ordinary Turing Machine with several tapes. Each tape has its own head for reading and writing. Initially, the input appears on tape 1, and the others start out blank. The transition function is modified to allow for reading, writing and moving on some or all of the tapes simultaneously. Formally, the transition function is defined as:

$\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$, where k is the number of tapes.

An additional head directions is modeled in Multitape Turing Machines set of possible movements, i.e. $\{S\}$, which indicates the Stationary movement. Multitape Turing Machines are shown to have equivalent powers to TMs. However, they are more convenient to represent parallel computation.

C. Universal Turing Machine (UTM)

A very powerful generalization of TMs is the Universal Turing Machine (UTM). When started on a tape containing the encoding of another Turing machine, call it *T*, followed by the input to *T*, a UTM produces the same result as *T* would when started on that input. Essentially a UTM can simulate the behavior of any Turing machine (including itself).

A UTM U can be formally defined as a Turing Machine which, when processing an input, it interprets this input as a description of another given Turing Machine, denoted M, concatenated with the description of an input data x for that machine. The function of U is to simulate the behavior of M processing input x. Thus mathematically a UTM can be defined by the equation: $U(M; x) = M(x)$.

From the modeling perspective, we argue that UTMs are very suitable for modeling the modularity property of cognition as discussed in Section II.

V. A Novel Formal Mathematical Theoretical Model for CR

The limitations of the computational models introduced earlier mandated the development of a new computational model. That new computational model should be capable of expressing the Cognition Cycle of a CR, in addition to being able to reflect the main cognition properties described earlier; the *modularity*, *parallel distributed processing*, and *evolutionary* nature of cognition. Hence, in this section we introduce a novel computational model, which can be considered as a merger between UTM and MTMs.

A. Novel Computational Model – Universal Multitape Turing Machine (UMTM)

A Universal Multitape Turing Machine is the merge of a Universal Turing Machine (UTM) and a Multitape Turing Machine (MTM). As in the normal UTM, has the capability of interpreting an input as a description of another given Turing Machine concatenated with the description of the input data for the given Turing Machine. However, this UTM has multiple tapes, with each tape equipped with its own head for reading and writing. Initially, the input also appears on tape 1, and the others start out blank.

This modified computational model is thought to be more adequate for simulating multiple machines or subsystems, each operating independently and autonomously on their corresponding inputs, i.e. the parallel distributed processing nature of the cognitive process.

Formally, a UMTM $\mho$ simulating the behavior of MTM $\mathcal{M}$ which is operating on the input alphabet Σ, can be modeled by $\mho(\mathcal{M}; \Sigma) = \mathcal{M}(\Sigma)$, where $\mathcal{M}$ is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, and Q, Σ, Γ are finite sets and

1. *Q is the set of states ,*
2. *Σ is the input alphabet not containing the blank symbol $\sqcup$,*
3. *Γ is the tape alphabet, where $\sqcup \in \Gamma$, and $\Sigma \subseteq \Gamma$,*
4. *$\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$ is the transition function*, where k is the number of tapes, and S is the Stationary movement of the head.
5. *$q_0 \in Q$, is the start state,*
6. *$q_{accept} \in Q$, is the accept state, and*
7. *$q_{reject} \in Q$, is the reject state*, where $q_{accept} \neq q_{reject}$

B. Novel Formal Cognition Cycle Model

Applying the UMTM computational model to the proposed cognition cycle presented in Figure 1, we get the following formal definition for the cognition cycle.

The cognition cycle of a CR denoted by the UMTM $\mho$ simulating the behavior of a MTM $\mathcal{M}$ operating on the input alphabet Σ, can be modeled as $\mho(\mathcal{M};\Sigma) = \mathcal{M}(\Sigma)$, where $\mathcal{M}$ is *the MTM Outermost*, a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, Q_{accept}, Q_{reject})$, where Q, Σ, Γ are finite sets and

1. *Q is the set of states* $= \{Goals \,\&\, Targets, Sensing, Perception, Gap \,\&\, Context, Constraints, Action, Performance, Learning, Co\text{-}operation$
2. *Σ is the input alphabet including the configuration of Turing Machine $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, but not containing the blank symbol $\sqcup$,*
3. *Γ is the tape alphabet, where $\sqcup \in \Gamma$, and $\Sigma \subseteq \Gamma$,*
4. *$\delta: Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R, S\}^k$ is the transition function*, where k is the number of tapes (a tape for every state) , and S is the Stationary movement of the head.
5. *$q_0 \in Q$, is the start state,*
6. *$q_{accept} \subseteq Q$, is the accept state, and*
7. *$q_{reject} \subseteq Q$, is the reject state*, where $q_{accept} \neq q_{reject}$

The δ transition function can be defined by the state diagram of Figure 1. For a complete description of UMTM $\mho$, every state belonging to Q, must be furthermore be modeled by means of another UMTM, till all the states have been fully specified, and all the cognitive functionalities have been mapped to the model. However, it is our aim in this work to present the idea of the computational model, through a thorough treatment of the cognitive processes of the mind, in addition to present the novel cognition cycle.

C. Modeling Evolution

Now, only one thing remains. How can we model mathematically the capability of evolving the states of the cycle? The work of Fred Cohen on theoretical computer virology will do the trick [32]. Fred Cohen's formalization is based on the notion of a viral set.

Fred Cohen's approach was to define a virus as a set containing elements, possibly many: the viral set. This viral set contains all possible different but equivalent forms (variants) of the viral program, obtained as the result of a computation. Evolution according to Cohen's formalization is the process according to which an element of a viral set is produced as a result of a transformation from different element of that set.

Inspired from Cohen's approach, we have formalized the evolution of the cognition cycle states; by defining the set containing all the possible different forms of cognitive programs *P*. However, we have generalized the approach by applying the concept of *closure*, as discussed in Section II.

We have for every Turing Machine $\mho$ belonging to the set of all possible Turing Machines Ψ,

$$[\forall \mho \in \Psi, \quad \forall p \, \forall i \in \mathbb{N}^*, p \in TP_M \text{ iff } p \in \Sigma_M^i$$

where p is a Turing program, and TP_M is a structure describing a Turing Machine program. This program is a finite sequence of symbols, each of them belonging to the reference alphabet for the tape.

And $[\forall \mho \in \Psi, \forall P \;\; P \in TS$ iff $\exists p \in P$ & $\forall p \in P,\; p \in TP_M$, where, TS is a non-empty set of Turing machine programs.

For all Turing machines $\mho$ and all non-empty sets or Turing programs, the pair $(\mho, P)$ is a Cognitive Set $\mathbb{C}$, if and only if, for each program $[\forall p \in P$, for all histories of the machine $\mho$, we have: For all time instants $t \in \mathbb{N}$ and cells $j \; of \; \mho$ if

[
1. the tape head is in front of cell j at time instant t and
2. $\mho$ is in its initial state at time instant t and
3. the tape cells starting at index j holds the program p,
]

then there exists a program $p' \in P$, at time instant $t' > t$ and at index j' such that

[
1. index j' is far enough from p position (start location j),
2. the tape cells starting at index j' hold the program p' and
3. at some time instant t'' such that $t < t'' < t', p'$ is written by $\mho$.
]

Formally, the above can be expressed as:

$$[\forall \mho \, \forall P \;\; (\mho, P) \in \mathbb{C} \iff [P \in TS] \& [\mho \in \Psi] \& [\forall p \in P \, [\forall H_M [\forall t \, \forall j$$

$$[1. \; cell_{\mho}(t) = j \;\&$$
$$2. \, state_{\mho}(t) = state_{\mho}(0) \;\&$$
$$3. \, (\text{tape\_content}_{\mho}(t, j), \ldots, \text{tape\_content}_{\mho}(t, j + |p| - 1)) = p]$$
$$\Rightarrow [\exists p' \in P \, [\exists t' > t \, [\exists j''$$
$$[1. \, [\, [j' + |p'| \leq j] or \, [j + |p| \leq j'] \,]$$
$$2. \, (\Delta_{\mho}(t', j'), \ldots, \Delta_{\mho}(t', j' + |p'| - 1)) = p'$$
$$3. \, [\exists t'' such \; that \; [t < t'' < t'] \&$$
$$[cell_{\mho}(t'') \in j', \ldots, j' + |p'| - 1]$$
$$]]]]]]]]$$

We have described the evolution of the states of the cognition cycle for any cognitive program just as Cohen described formally his evolving virus. That completes the formal definition of the cognition cycle.

Note that the subsumption architecture is implicitly modeled into the cognition cycle described in Section III.

D. Formal Cognitive Radio Model

A formal mathematical definition for CR is now a straightforward task based on the formal model developed for the cognition cycle: *"Cognitive Radio is computationally modeled by a Universal Multitape Turing Machine (UMTM) $\mho$ characterizing a cognition cycle as defined by Definition 1, and running a Turing program $p \in P$ such that:*

$$[\forall \mho\ \forall P\ \ (\mho, P) \in \mathbb{C}\ \Leftrightarrow [P \in\ TS] \& [\mho \in \Psi\ ] \& [\forall p \in P\ [\forall H_M [\forall t\ \forall j$$
$$[1.\ \ cell_{\mho}(t) =\ j\ \&$$
$$2.\ state_{\mho}(t) =\ state_{\mho}(0)\ \&$$
$$3.\ (\mathrm{tape}_{\mathrm{content}_{\mho}}(t, j), \dots, \mathrm{tape}_{\mathrm{content}_{\mho}}(t, j + |p| - 1)) = p]$$
$$\Rightarrow [\exists p' \in P\ [\exists t' > t\ [\exists j''$$
$$[1.\ [\ [j' +\ |p'| \leq j] or\ [j + |p| \leq j']\ ]$$
$$2.\ (\Delta_{\mho}(t', j'), \dots, \Delta_{\mho}(t', j' + |p'| - 1)) = p'$$
$$3.\ [\exists t'' such\ that\ [t <\ t'' <\ t'] \&$$
$$[cell_{\mho}(t'') \in\ j', \dots, j' +\ |p'| - 1]$$
$$]]]]]]]]$$

VI. Model Generality and Openness

Mitola defined CR as [2,3]: “A radio that employs model based reasoning to achieve a specified level of competence in radio-related domains.” He demonstrated that the CR follows OODA cognition cycle reflecting the sequence of steps, and the states employed by any CR during its operation [33].

Haykin [34] gives another comprehensive definition for CR focusing on three on-line cognitive tasks, namely, radio-scene analysis, channel identification, and transmit-power control and dynamic spectrum management.

Other definitions – like those of the FCC, SDR Forum, IEEE – are from an operational or application-oriented view. Taking Mitola as an example, we find that his OODA cycle can be directly incorporated into our proposal as shown in Figure 2.

Mitola	**→ Our Proposal**
O Observe	→ *Sensing*
O Orient	→ *Perception & Attention + Gap and Context + Constraints*
D Decide	→ Decision is an implicit feature in our proposal. An intelligent agent is mandatory for the cognitive operation of CR.
A Act	→ *Action*

Fig. 2. OODA Loop Incorporated into our Cognition Cycle Proposal

We offer finer levels of abstraction based on our analysis of the human cognitive process. In addition, we do have some additional blocks that weren't present in Mitola's cycle, like *Performance*, *Co-Operation*, and *Goals & Targets*.

The point to make is that our model, from the modeling perspective, is capable of accommodating any existing definition for CR due to its generality, high expressive power, and openness. All functions specified by Mitola, and others can be mapped directly to a certain state in our proposed cognition cycle. However, we do hold the position that any specifications for cognition should be based on a formal theoretical model of the real cognitive processes of the mind. And that's what we have endeavored ourselves to do.

VII. Conclusion and Future Work

In this paper we have presented a novel formal mathematical model of CR. For the first time, to our knowledge, formalization of CR has been proposed, thus putting down a preliminary effort towards developing a formal rigorous treatment for all future CR aspects. This manuscript serves as a first step towards this goal. We have used cognitive sciences, cognitive neuroscience and artificial intelligence as the theoretical base for our work, in order to characterize the true *cognitive* capabilities. A novel computational theoretic model was developed to capture central cognition aspects like modularity, parallel distributed processing, and evolution.

This model is shown to be flexible enough to accommodate all current views and definitions of CR. Being based on UMTM, a novel computational model developed specifically to model the peculiar nature of cognition, it opens up further mathematical treatment on the computability and complexity of cognitive processes. The UMTM is a computational model, which happens to possess adequate expressive powers; which makes it suitable for modeling cognition inside CR. The UMTM is capable of expressing modular, parallel designs, in addition to being able to express the concepts of evolutions and hierarchy, systems and sub-systems, or modules and sub-modules which themselves can consists of other modules. That recursive nature, in addition to the ability of modeling each module or CP, as a machine, which functions by applying a certain input to that machine, is what renders the UMTM as a good model for representing the CR's Cognition Cycle.

Cognition as we understand it includes – among others – some fundamental tasks like learning, adaptation, and intelligence. Intelligence through an intelligent agent is foreseen as an implicit mandatory component of a CR.

Future work includes the analysis on the computability and complexity of some of our proposed modules of cognition. This serves in answering some of the challenging open questions on the feasibility of implementing full cognition capabilities in radio system to attain true *cognitive* behavior.

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