A Damped Guass-Newton Method for the Generalized Linear Complementarity Problem

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Abstract—**In this paper, we consider the generalized linear complementarity problem (GLCP) over an affine subspace. To this end, we first reformulate the GLCP as a system of nonsmooth equation via the Fischer function. Based on this reformulation, the famous damped Gauss-Newton (DGN) algorithm is employed for obtaining its solution, and we show that the DGN algorithm is quadratically convergent without nondegenerate solution. Some numerical experiments of the algorithm are also reported in this paper.**

Keywords- GLCP; DGN algorithm; quadratical convergence

I. INTRODUCTION

The generalized linear complementarity problem, denoted by the GLCP, is to find a vector $(x^*,y^*)\in \mathrm{R}^{2n}$ such that

$$Mx^* - Ny^* \in \mathrm{K}, x^* \geq 0, y^* \geq 0, (x^*)^T y^* = 0, \tag{1}$$

where $M, N \in R^{m\times n}$ are two given matrices, and $\mathrm{K} = \{Qz+q \mid z\in R^l\}(Q\in R^{m\times l}, q\in R^m)$ is an affine subspace in R^m.

The GLCP is a special case of the extended linear complementarity (XLCP) which was firstly introduced by Mangasarian and Pang ([1]).The generalized complementarity problem plays a significant role in economics equilibrium problems, noncooperative games, traffic assignment problems, engineering and operation research, and of course in optimization problems ([2]).

For the GLCP, many effective methods have been proposed in recent years ([3]). Zhang et al. ([4]) reformulate the GLCP as unconstrained smooth optimization problem via Fischer function, and design a Newton-type algorithm for solving it. D.S.Bart and M.D.Bart ([5]) develop a double description method to find all its solutions, and show the general problem is a NP-hard problem. Different from the algorithms listed above, in this paper, we equivalently reformulate the GLCP as a system of nonsmooth equations via the Fischer function. Based on this reformulation, we propose a damped Gauss-Newton algorithm to solve this system. We show that the algorithm is quadratically convergent under milder hypotheses.

We end this section with some notations used in this paper. The inner product of vectors $x, y \in R^n$ is denoted by $x^T y$. Let $\|\cdot\|$ denote 2-norm of vectors in Euclidean space.The transposed Jacobian $F'(x)$ of a vector-valued function $F(x)$ is denoted by $\nabla F(x)$.For simplicity, we use (x, y, z) for column vector $(x^T, y^T, z^T)^T$.For $a\in R^n, D_a = diag(a)$ denotes the diagonal matrix in which the i -th diagonal element is a_i.

II. EQUIVALENT STATEMENTS OF GLCP

In this section, we will give some equivalent statements relative to the solution of the GLCP. First, the following result is straightforward.

Theorem 2.1 (x^*,y^*) is a solution of the GLCP if and only if there exists $z^*\in R^l$ such that

$$\begin{cases} Mx^* - Ny^* - Qz^* - q = 0 \\ x^* \geq 0, y^* \geq 0, (x^*)^{\mathrm{T}} y^* = 0 \end{cases}$$

To propose a quadratically convergent algorithm for the solution of the GLCP, we now formulate the GLCP as a system of equations via the Fischer function ([6]) $\phi: R^2 \to R^1$ defined by

$$\phi(a,b) = \sqrt{a^2+b^2} - a - b, \text{ for } a,b\in R$$

A basic property of this function is that

$$\phi(a,b) = 0 \Leftrightarrow a\geq 0, b\geq 0, ab = 0$$

For arbitrary vectors $a,b\in R^n$, we define a vector-valued function as follows

$$\Phi(a,b)=\begin{pmatrix}\phi(a_1,b_1)\\ \phi(a_2,b_2)\\ \cdots\cdots\\ \phi(a_n,b_n)\end{pmatrix}.$$

Obviously, $\Phi(a,b)=0 \Leftrightarrow a\ge 0, b\ge 0, a^{\mathrm{T}}b=0$

Combining this conclusion with Theorem 2.1, we can establish the following equivalent formulation of the GLCP.

Theorem 2.2 (x^*,y^*) is a solution of the GLCP if and only if there exists $z^*\in R^l$ such that

$$\begin{cases} Mx^*-Ny^*-Qz^*-q=0\\ \Phi(x^*,y^*)=0\end{cases}. \tag{2}$$

From the analysis above, we define a vector-valued function $\Psi: R^{2n+l}\to R^{m+n}$ and a real-valued function $f: R^{2n+l}\to R$ as follows:

$$\Psi(x,y,z):=\begin{pmatrix}Mx-Ny-Qz-q\\ \Phi(x,y)\end{pmatrix}, \tag{3}$$

$$f(x,y,z):=\frac{1}{2}\Psi(x,y,z)^{\mathrm{T}}\Psi(x,y,z)=\frac{1}{2}\|\Psi(x,y,z)\|^2. \tag{4}$$

Combining (2) with (4), the following conclusion is obvious.

Theorem 2.3 (x^*,y^*) is a solution of the GLCP if and only if $f(x^*,y^*,z^*)=0$.

III. Some Definitions and Basic Results

In this section, we first review some definitions which will be used in the sequel.

For a locally Lipschitzian mapping $H: R^n\to R^m$, we let $\partial H(x)$ denote the Clarke's generalized Jacobian of $H(x)$ at $x\in R^n$ which can be expressed as the convex hull of the set $\partial_B H(x)$ ([7]), where

$$\partial_B H(x)=\left\{V\in R^{m\times n}\,|\, V=\lim_{x^k\to x} H'(x^k)\right\},$$

$H(x)$ is differentiable at x^k for all k. The following definitions are due to Qi and Sun ([8]).

Definition 3.1 A locally Lipschitz continuous vector valued function $\Theta: R^n\to R^m$ is said to be semismooth at $x\in R^n$, if the limit

$$\lim_{\substack{V\in\partial H(x+th')\\ h'\to h, t\downarrow 0}}\{Vh'\}$$

exists for any $h\in R^n$

Definition 3.2 The function $H: R^n\to R^m$ is said to be strongly semismooth at x if H is semismooth at x and for any $V\in\partial H(x+h), h\to 0$, it holds that

$$H(x+h)-H(x)-Vh=O(\|h\|^2).$$

Next, we discuss the differential properties of Ψ and f defined by (3) and (4). The function $\Psi(x,y,z)$ is not differentiable everywhere with respect to $(x,y,z)\in R^n\times R^n\times R^l$. However, it is locally Lipschitzian continuous vector valued function, and there has a nonempty generalized Jacobian in the sense of Clarke ([9]). In the following, we give an approach to calculate an element of $\partial\Psi(x,y,z)$.From Proposition 3.1 in [10], we give the following result.

Proposition 3.1 For $x,y\in R^n$, choose $v\in R^n$ such that $v_i\ne 0$ for any index i with $x_i=0$ and $y_i=0$. Let $W=(D_a,D_b)$, where

$$a_i=\frac{x_i}{\sqrt{x_i^2+y_i^2}}-1, b_i=\frac{y_i}{\sqrt{x_i^2+y_i^2}}-1, if x_i^2+y_i^2>0;$$

$$a_i=\frac{1}{\sqrt{v_i^2+1}}-1, b_i=\frac{v_i}{\sqrt{v_i^2+1}}-1, if x_i^2+y_i^2>0.$$

Then $W\in\partial\Phi(x,y)$, or more precisely, $W\in\partial_B\Phi(x,y)$.

Changing v, we will obtain a different element of $\partial_B\Phi(x,y)$. In our code, we choose to set $v_i=0$ if $x_i^2+y_i^2>0$, otherwise. Thus, an element $V\in\partial\Psi(x,y,z)$ can be calculated as

$$V=\begin{pmatrix}M & -N & -Q\\ D_a & D_b & 0\end{pmatrix}$$

where D_a and D_b are defined in Proposition 3.1. It is easily seen that, when $m=n+l$, V is square.

We summarize the differential properties of Ψ and f in the following lemma, and its proof can be found in [10].

Lemma 3.1 For the vector-valued function Ψ and real-valued function f defined by (3) and (4), the following statements hold:

(a) Ψ is strongly semismooth.

(b) f is continuously differentiable, and its gradient at a point $(x,y,z)\in R^n\times R^n\times R^l$ is given by $\nabla f(x,y,z)=V^{\mathrm{T}}\Psi(x,y,z)$, where V is an arbitrary element belonging to $\partial\Psi(x,y,z)$.

IV. STATIONARY POINT AND NONSINGULARITY CONDITIONS

To establish a quadratic convergence rate of our algorithm proposed in the next section, we need to study the conditions under which every element of the generalized Jacobian $\partial\Psi(\omega)$ is full row rank at a solution point ω^* of the equation $\Psi(\omega)=0$. First, we give the needed definition ([11]).

Definition 4.1 Given two matrices $M,N\in R^{m\times n}$, we say that M,N has the row P-property if it satisfies the condition

$$(M^{\mathrm{T}}\mu, N^{\mathrm{T}}\mu)\neq 0, \mu\in R^n \Rightarrow \exists (M^{\mathrm{T}}\mu)_{i_0}\neq 0,$$
$$(N^{\mathrm{T}}\mu)_{i_0}\neq 0, (M^{\mathrm{T}}\mu)_{i_0}(N^{\mathrm{T}}\mu)_{i_0}>0$$

Theorem 4.1 Suppose that M,N have the row P-property and rank$[M,N]=m$, then for any $V\in\partial\Psi(\omega)$, V is of full row rank. Moreover, when $m=n+l$, V is nonsingularity.

Proof: Assume that V is not of full row rank. Then there is a nonzero vector $(u,v)\in R^{m+n}$ such that $V^{\mathrm{T}}(u,v)=0$, i.e.,

$$M^{\mathrm{T}}u+D_a v=0, -N^{\mathrm{T}}u+D_b v=0, Q^{\mathrm{T}}u=0. \tag{5}$$

This implies that

$$(M^{\mathrm{T}}u)_i(N^{\mathrm{T}}u)_i=-(D_a v)_i(D_b v)_i=-a_i b_i v_i^2, \quad i=1,2,\cdots,n. \tag{6}$$

By $a_i\le 0, b_i\le 0$ and (6), we have

$$(M^{\mathrm{T}}u)_i(N^{\mathrm{T}}u)_i\le 0, i=1,2,\cdots,n. \tag{7}$$

Suppose that $(M^{\mathrm{T}}u)(N^{\mathrm{T}}u)=0$, by rank $[M,-N]=m$, we get $u=0$. Since $(u,v)\neq 0$ and $u=0$, we have $v\neq 0$. Without loss of generality, we assume that $v_{i_1}\neq 0$, from (5) and $u=0$, we obtain $D_a v=0$, $D_b v=0$, and $a_{i_1}=b_{i_1}=0$, which contradicts that a_i, b_i defined in Proposition 3.1. So

$$(M^{\mathrm{T}}u)(N^{\mathrm{T}}u)\neq 0. \tag{8}$$

By (8) and the row P-property of M,N, we have that there exists i_0 such that $(M^{\mathrm{T}}u)_{i_0}(N^{\mathrm{T}}u)_{i_0}>0$, which is a contradiction to (7).

So, V is of full row rank. Obviously, when $m=n+l$, V is nonsingularity.

By Theorem 2.3, we know that a point (x^*,y^*) is a solution of the GLCP if and only if $f(x^*,y^*,z^*)=0$, or equivalently, $\omega^*=(x^*,y^*,z^*)$ is a global minimizer with zero objective function value of the unconstrained optimization problem

$$\min_{\omega\in R^{n+n+l}} f(\omega). \tag{9}$$

It is necessary to establish conditions, which guarantees that every stationary point of (9) solves the GLCP. The following theorem gives a suitable condition.

Theorem 4.2 Let $\omega^*=(x^*,y^*,z^*)$ is a stationary point of (9), if M,N has the row P-property and rank$[M,N]=m$, then (x^*,y^*) is a solution of the GLCP.

Proof: Since ω^* is a stationary point of (9), then $\nabla f(\omega^*)=0$, i.e., $(V^*)^{\mathrm{T}}\Psi(\omega^*)=0$. By Theorem 4.1, we have V^* is of full row rank, so $\Psi(\omega^*)=0$. Moreover, (x^*,y^*) is a solution of the GLCP.

V. ALGORITHM AND CONVERGENCE

In this section, we formally state a damped Gauss-Newton (DGN) algorithm, which is similar to the algorithm in [12].

For convenience, let $\omega^k=(x^k,y^k,z^k)$ in the sequel.

DGN Algorithm

Step1. Let $\omega^0\in R^{n+n+l}$ and $0<\delta<1$ be given.

Step2. If $\nabla f(\omega^k)=0$, stop; otherwise, go to step3.

Step3. Choose an element $V^k\in\partial\Psi(\omega^k)$, set

$$p^k=((V^k)^{\mathrm{T}}V^k+\lambda_k I)^{-1}\nabla f(\omega^k),$$

where $\lambda_k=f(\omega^k)$.

Step4. Let α_k be the largest element in the set $\Omega=\{1,1/2,\cdots\}$ such that

$$f(\omega^k)-f(\omega^k-\alpha_k p^k)\ge\delta\alpha_k\nabla f(\omega^k)^{\mathrm{T}}p^k.$$

Step5. Set $\omega^{k+1}=\omega^k-\alpha_k p^k$, $k=k+1$, go to Step 2.

Lemma 5.1 For any $\omega=(x,y,z)\in R^{2n+l}$. Suppose that $\nabla f(\omega)\neq 0$. Then, given $\lambda>0$, the direction p given by $(V^{\mathrm{T}}V+\lambda I)p=\nabla f(\omega)$ is an ascent direction for $f(\omega)$, it is that, $\nabla f(\omega)^{\mathrm{T}}p>0$.

Proof: Obviously, there exist constants $\gamma_1\ge 0$ and $\gamma_2\ge 0$ such that

$$\gamma_1\|r\|^2\le r^{\mathrm{T}}(V^{\mathrm{T}}V)r\le\gamma_2\|r\|^2, \forall r\in R^n.$$

Thus, for any $r\in R^n$,

$$(\gamma_1+\lambda)\|r\|^2\le r^{\mathrm{T}}(V^{\mathrm{T}}V+\lambda I)r\le(\gamma_2+\lambda)\|r\|^2. \tag{10}$$

Since $\nabla f(\omega)=V\cdot\Psi(\omega)\neq 0$, then $V\neq 0$, and $V^{\mathrm{T}}V\neq 0$.

By $(V^{\mathrm{T}}V+\lambda I)p=\nabla f(\omega)$, we have $p\neq 0$. If we let $r=p$ in (10), we get

$$p^{\mathrm{T}}\nabla f(\omega)=p^{\mathrm{T}}(V^{\mathrm{T}}V+\lambda I)p\geq(\gamma_1+\lambda)\|p\|^2>0.$$

It follows that $\nabla f(\omega)\cdot p>0$ and that p is an ascent direction for $f(\omega)$.

According to Lemma 5.1, it is easy to say that $-p^k$ is a descent direction of $f(\omega)$ at ω^k and the DGN Algorithm is well defined. Obviously, if $\nabla f(\omega^k)=0$, then ω^k is a stationary point of problem (9). Thus, (x^k,y^k) is a solution of the GLCP under suitable conditions.

Theorem 5.1 Let $\{\omega^k\}$ be the sequence determined by the DGN Algorithm, then either $\{\omega^k\}$ terminates at a stationary point of $f(\omega)$, or else every accumulation point of $\{\omega^k\}$,if it exists, is a stationary point of $f(\omega)$.

Proof: The first assertion is obvious, we prove only the second.

Let $\omega^*\in R^{n+n+l}$ be an accumulation point of $\{\omega^k\}$, i.e., there exists an infinite subsequence of $\{\omega^k\}$ converges to ω^*. Without loss of generality, we assume that $\{\omega^k\}$ converges to ω^*. Since the subdifferential is upper semicontinuous, the sequence $\{V^k\}$ is bounded. With loss of generality, we may assume that $\{V^k\}\to V^*$. Since the sequence $\{f(\omega^k)\}$ is decreasing and bounded from below, $\{f(\omega^k)\}\to f(\omega^*)$. Therefore,

$$\{\nabla f(\omega^k)\}=\{V^k\Psi(\omega^k)\}\to V^*\Psi(\omega^*)=\nabla f(\omega^*),$$
$$\{(V^k)^{\mathrm{T}}V^k+\lambda_k I\}=\{(V^k)^{\mathrm{T}}V^k+f(\omega^k)I\}$$
$$\to(V^*)^{\mathrm{T}}V^*+f(\omega^*)I=(V^*)^{\mathrm{T}}V^*+\lambda_* I,$$

then, we have $\{p^k\}\to p^*$.

Suppose that $\nabla f(\omega^*)\neq 0$, then $V^*\neq 0$, $\Psi(\omega^*)\neq 0$, and $\lambda_*=f(\omega^*)=\frac{1}{2}\|\Psi(\omega^*)\|^2>0$, $(V^*)^{\mathrm{T}}V^*+\lambda_* I$ is positive definite. Thus,

$$\nabla f(\omega^*)p^*=\nabla f(\omega^*)((V^*)^{\mathrm{T}}V^*+\lambda_* I)^{-1}\nabla f(\omega^*)>0.$$

Let α^* be the largest element α in the set $\Omega=\{1,1/2,\ldots\}$ such that

$$f(\omega^*)-f(\omega^*-\alpha p^*)\geq\delta\alpha\nabla f(\omega^*)^{\mathrm{T}}p^*.$$

By the continuity of f, for k sufficiently large, we have

$$f(\omega^k)-f(\omega^k-\alpha^* p^k)\geq\delta\alpha^*\nabla f(\omega^k)^{\mathrm{T}}p^k.$$

From the stepsize rule of α^k, we know that

$$\begin{aligned}&f(\omega^k)-f(\omega^{k+1})\\&=f(\omega^k)-f(\omega^k-\alpha^k p^k)\\&\geq f(\omega^k)-f(\omega^k-\alpha^* p^k)\\&\geq\delta\alpha^*\nabla f(\omega^k)^{\mathrm{T}}p^k.\end{aligned}\tag{11}$$

Taking the limit on both side of (11), we get

$$0=f(\omega^*)-f(\omega^*)\geq\delta\alpha^*\nabla f(\omega^*)^{\mathrm{T}}p^*.$$

But it is impossible since $\nabla f(\omega^*)p^*>0$.

Therefore, $\nabla f(\omega^*)=0$, i.e., ω^* is a stationary point of f.

The next result follows immediately from Theorem 4.2 and Theorem 5.1.

Theorem 5.2 Let $\{\omega^k\}$ be the sequence determined by the DGN Algorithm, $\omega^*=(x^*,y^*,z^*)$ be an accumulation point of $\{\omega^k\}$. Suppose that M,N has the row P-property and rank$[M,N]=m$, then (x^*,y^*) is a solution of GLCP.

Next, we prove the quadratic convergence of DGN Algorithm.

Theorem 5.3 Let $\{\omega^k\}$ be the sequence determined by the DGN Algorithm, $\{\omega^*\}$ be an accumulation point of $\{\omega^k\}$. Suppose that M,N has the row P-property and rank $[M,N]=m$, then, we have the sequence $\{\omega^k\}$ converges to $\{\omega^*\}$ quadratically.

Proof: Since ω^* is an accumulation point of $\{\omega^k\}$, there exists an infinite subsequence of $\{\omega^k\}$ converges to ω^*. Without loss of generality, we assume that $\{\omega^k\}$ converges to ω^*.

By Theorem 5.2, we have (x^*,y^*) is a solution of GLCP, so $\Psi(\omega^*)=0$, $f(\omega^*)=0$. By the proof of Theorem 5.1, we obtain

$$\{\lambda_k\}=\{f(\omega^k)\}\to f(\omega^*)=0,\{V^k\}\to V^*.$$

Then, there exist constants $\xi_1>0,\xi_2>0$ such that for all k sufficiently large and for any $V^k\in\partial\Psi(\omega^k)$

$$\|\lambda_k\|\leq\xi_1,\|(V^k)^{\mathrm{T}}\|\leq\xi_2.\tag{12}$$

Moreover, there exists a constant $\xi>0$ such that

$$\|((V^k)^{\mathrm{T}}V^k+\lambda_k I)^{-1}\|\leq\xi.\tag{13}$$

By Lemma 3.1, we have $\Psi(\omega^k)$ is strongly semismooth, and by the definition of strong semismoothness, we have

$$\left\|\Psi(\omega^k)-\Psi(\omega^*)-V^k(\omega^k-\omega^*)\right\| = O(\left\|\omega^k-\omega^*\right\|^2).$$

So, there exists ξ_3 such that

$$\frac{\left\|\Psi(\omega^k)-\Psi(\omega^*)-V^k(\omega^k-\omega^*)\right\|}{\omega^k-\omega^*} \to \xi_3. \quad (14)$$

Similarly to the proof of Theorem 3.1 in [10], we can show that $\alpha_k = 1$ for sufficiently large k and, $\omega^{k+1} = \omega^k - p^k$. Thus,

$$\begin{aligned}
&((V^k)^{\mathrm{T}}V^k+\lambda_k I)(\omega^{k+1}-\omega^*)\\
&=((V^k)^{\mathrm{T}}V^k+\lambda_k I)(\omega^k-\omega^*-p^k)\\
&=((V^k)^{\mathrm{T}}V^k+\lambda_k I)(\omega^k-\omega^*)-\nabla f(\omega^k)\\
&=(V^k)^{\mathrm{T}}V^k(\omega^k-\omega^*)+\lambda_k(\omega^k-\omega^*)-\nabla f(\omega^k)\\
&=-[\nabla f(\omega^k)-(V^k)^{\mathrm{T}}V^k(\omega^k-\omega^*)]+\lambda_k(\omega^k-\omega^*) \quad (15)\\
&=-[(V^k)^{\mathrm{T}}\Psi(\omega^k)-(V^k)^{\mathrm{T}}\Psi(\omega^*)\\
&-(V^k)^{\mathrm{T}}V^k(\omega^k-\omega^*)]+\lambda_k(\omega^k-\omega^*)\\
&=-(V^k)^{\mathrm{T}}[\Psi(\omega^k)-\Psi(\omega^*)-V^k(\omega^k-\omega^*)]\\
&+\lambda_k(\omega^k-\omega^*).
\end{aligned}$$

Since the continuity of f and $f(\omega^*)=0$, there exists a constant $\xi_4 > 0$ such that

$$\left\|\lambda_k\right\| = \left\|f(\omega^k)\right\| = \left\|f(\omega^k)-f(\omega^*)\right\| \le \xi_4\left\|\omega^k-\omega^*\right\|. \quad (16)$$

By (12) $\to$ (16), we have

$$\begin{aligned}
&\left\|\omega^{k+1}-\omega^*\right\|\\
&\le \xi\xi_2\left\|\Psi(\omega^k)-\Psi(\omega^*)-V^k(\omega^k-\omega^*)\right\|+\xi\xi_4\left\|\omega^k-\omega^*\right\|^2\\
&\to \xi\xi_2\xi_3\left\|\omega^k-\omega^*\right\|^2+\xi\xi_4\left\|\omega^k-\omega^*\right\|^2\\
&=\eta\left\|\omega^k-\omega^*\right\|^2,
\end{aligned}$$

where $\eta = \xi\xi_2\xi_3 + \xi\xi_4$ is a constant.

Therefore, we have $\{\omega^k\}$ converges to ω^* quadratically.

VI. Computational Experiments

In the following, we will implement the DGN Algorithm in Matlab and run it on Pentium IV computer.

Example 6.1 (*Murty 1988*). n variables,

$$M=\begin{pmatrix}1&2&2&\cdots&2\\0&1&2&\cdots&2\\0&0&1&\cdots&2\\\cdots&\cdots&\cdots&\cdots&\cdots\\0&0&0&\cdots&1\end{pmatrix}, N=\begin{pmatrix}1&0&0&\cdots&0\\0&1&0&\cdots&0\\0&0&1&\cdots&0\\\cdots&\cdots&\cdots&\cdots&\cdots\\0&0&0&\cdots&1\end{pmatrix},$$

$$Q=0, q=(1,1,\cdots,1)^{\mathrm{T}}$$

We take $x^0=(1,1,\cdots,1)^{\mathrm{T}}$ as our starting point. The solution is $x^*=(0,0,\cdots,1)^{\mathrm{T}}$, $y^*=(1,1,\cdots,0)^{\mathrm{T}}$. The numerical results for this test problem can be found in Table 1. For this problem, Harker and Pang ([13]) used the damped-Newton method (DNA), and Zhang et al. ([14]) used the Newton-type method (NTA). The results for the above two methods and several values of the dimensions n are summarized in Table 2. From Table 1 and Table 2, we can conclude that our algorithm excels the other two methods listed above.

TABLE 1.
Numerical results of our algorithm for Example 6.1

Dimension	8	16	32	64	128
Iter.num	7	12	17	41	82

TABLE 2.
Numerical results by DNA, NTA

Dimension	8	16	32	64	128
DNA iter.num	9	20	72	208	>300
NTA iter.num	13	12	18	99	99

Example 6.2 This example is LCP used by Noor ([14]). n variables,

$$M=\begin{pmatrix}1&0&0&\cdots&0\\0&1&0&\cdots&0\\0&0&1&\cdots&0\\\cdots&\cdots&\cdots&\cdots&\cdots\\0&0&0&\cdots&1\end{pmatrix}, N=\begin{pmatrix}4&-2&0&\cdots&0\\1&4&-2&\cdots&0\\0&1&4&\cdots&0\\\cdots&\cdots&\cdots&\cdots&\cdots\\0&0&0&\cdots&-2\\0&0&0&\cdots&4\end{pmatrix},$$

$$Q=0, q=(1,1,\cdots,1)^{\mathrm{T}}.$$

Table 3 list the results for this example with initial point $y^0=-N^{-1}q$ for different dimensions n and parameter $\delta=0.5$. Compared with the results of Table 4.2 in [13], we can conclude that our algorithm excels methods in [13].

TABLE 3.
Numerical results of our algorithm for Example 6.2

Dimension	10	20	50	80	100	200
Iter.num.	4	4	4	4	4	4
$\nabla f(\omega^*)$	0	0	0	0	0	0

Acknowledgment

The authors would like to thank the referees for their very helpful suggestions which greatly improved this paper. This work was supported by the Natural Science Foundation China (Grant No. 11271226), the Projects for Reformation of Chinese Universities Logistics Teaching and Research (JZW2012065), and the national college students' innovation and entrepreneurship training program (201210452027).

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