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Some Integral Properties of Signal Envelope in Weibull Fading Channel

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Abstract—**In this paper the Weibull fading channel model is described and some integral characteristics of Weibull distribution are analyzed. For analytical and numerical evaluation of system performance, the Weibull probability density functions (pdf) are analyzed like particular solutions of corresponding differential equation, while the existence of singular solution is considered and analyzed under different conditions. It is also shown that the envelope of probability density function curves family is a straight line, for all values of varying parameters. This new expression can be used during mobile system designing.**

Keywords-Weibull fading channel, probability density function, singular solution

I. Introduction

The Weibull distribution is usually used in communication systems to model fading channels in wireless communications [1] describing statistics of mobile radio transmission in complex medium such as the urban environment and also in radar systems modeling the dispersion of the received signals level produced by some types of clutters [2]. It is shown that the Weibull distribution can model different propagation conditions [3], providing more flexibility and higher accuracy in matching some experimental data in comparison with the commonly adopted distributions [4, 5]. The Weibull distribution is also applied in industrial engineering representing manufacturing and delivery times, in reliability engineering and in failure analysis [1]. The Weibull fading model confirms experimentally attained fading measurements for both indoor and outdoor propagation environment [6-8]. In this paper some new statistical properties of Weibull distribution are contributed.

The Weibull pdf is a function of several variables, such as Weibull shape parameter, describing fading depth, and Weibull scale parameter, related to the moments and fading depth [1-3]. In the process of determination and analysis of Weibull distribution integral characteristics, one of the pdf variables is treated as a parameter, while the others are set to certain constant values of interest in practice. In this way, the pdf curves family is obtained. The analysis of the position of the maximums for curves family can be performed analytically, using the first derivative of function, and also numerically. The same procedure is repeated for the case when the second variable is treated as a parameter, and the others are set to constant values. This procedure gives a new pdf curves family, while the maximum position is determined by a new envelope. In papers [9-12] the position of maximums and the analytical expression for the integral characteristic of Nakagami-*m* distribution are determined, while some integral properties of Ricean distribution are presented in [13]. Some statistical characteristics of signal envelope in correlated Weibull fading channel are given in [14, 15].

Using described analysis two series of pdf curves families are obtained, one for case when the shape factor is treated as a parameter and another when the scale factor is treated as a parameter. For each of pdf curves families, the equation of the envelope of curves maximums is considered. In both cases these envelopes are straight lines, in the logarithmic scale. The direction coefficients and values on ordinate-axis are determined analytically and numerically, for both the envelopes. Also, in both cases, the differential equations describing the process dynamics are determined, whereby the envelopes of the pdf curves families represent their singular solutions and pdfs represent their particular solution. The numerical modeling and some simulations using MATLAB and *Mathematica* are also given.

II. The Procedure of Determining the Integral Characteristics of Distribution

The previous analysis of probabilistic models for the description of the received signal envelope, depending on the individual relevant parameters for specific propagation conditions, resulted in the idea to focus the subject of research at the request of the integral description of phenomena that affect the radio signal transmission. We seek to determine analytical expressions for the description of the boundary receiving conditions that could be useful in the wireless receiver structures design.

For the considered Weibull model, for a fixed value of the fading depth parameter, the pdf of signal envelope, depending on the received signal level, is analyzed for different values of parameter related to the moments. Then the parameter related to the moments is treated as a parameter, while the values of the received signal level are varied. In such a way, two series of families of pdf curves are obtained and for each of them an equation of the envelope of curves maximums is contributed. In both cases these envelopes are straight lines, in the logarithmic scale, whose direction coefficients and values on ordinate-axis are determined analytically and numerically. The same procedure is repeated for different values of the fading depth parameter.

The analytical and numerical results obtained in this paper indicate the existence of differential equation whose particular solution is the pdf of the observed distribution. The envelope of maximums of the family of pdf curves represents the singular solution of differential equation [16] describing the dynamics of the process of signal transmission in the fading channel.

III. Weibull Fading Channel Model

The Weibull distribution is suitable for describing statistics of mobile radio transmission in complex media such as the urban environment [1]. In practice, it has proved very useful due to an easy manipulation and a wide range of applicability of various approximations [4]. The Weibull distribution is described by the pdf [4]:

$$p_z\left(z,\Omega\right)=\frac{\beta}{\Omega}z^{\beta-1}\exp\left(-\frac{z^{\beta}}{\Omega}\right) \tag{1}$$

where z presents Weibull random variable, β presents the shape parameter describing fading depth and parameter Ω presents the scale parameter of distribution, defined as:

$$\Omega=\mathrm{E}\left[z^{\beta}\right] \tag{2}$$

The Weibull shape parameter (fading parameter) can take values between 0 and ∞. The Weibull scale parameter Ω is a positive number related to the moments and the fading parameter.

The Weibull distribution is related to a number of other probability distributions, interpolating between the exponential distribution (for β=1) and the Rayleigh distribution (for β=2).

The Weibull distribution is very useful and flexible statistical model for describing the received signal envelope fluctuations in both indoor and outdoor propagation environment. The useful property of Weibull distribution is that the k^{th} ($k\in\mathbb{N}$) power of a Weibull distributed random variable with parameters (β, Ω) is also Weibull distributed random variable with parameters (β / k, Ω). This property is very useful in error rate analysis of digital modulations in Weibull fading channel, since the received signal-to-noise ratio (SNR) will also have a Weibull distribution.

IV. Integral Characteristics of Weibull Distribution

The graphic presentation of the dependence of Weibull pdf versus the received signal level, for a fixed value of the fading depth parameter and different values of Ω, in logarithmic scale, is shown in Fig. 1.

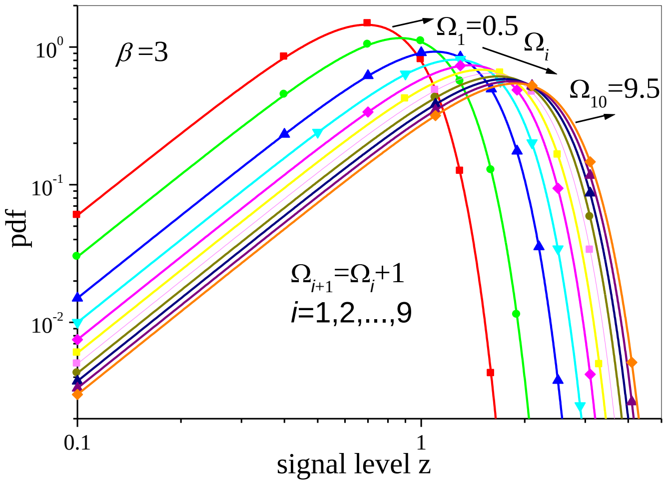

Figure 1. Weibull pdf versus z for different values of Ω and β=3

The analysis of this dependence shows that the increase of parameter Ω results in smaller values of pdf maximums, reached for higher values of received signal level, as shown in Fig. 1.

It can be concluded that all the maximums lie along a straight line, so the envelope of maximums is a straight line, in a logarithmic scale. In order to determine the values of the received signal level when the maximums are reached, as well as the values of maximums and a direction coefficient of the envelope, the first derivative of (1) relative to received signal level is determined. Equalization with zero obtains:

$$z_{p\max}=\sqrt[\beta]{\frac{\Omega\left(\beta-1\right)}{\beta}} \tag{3}$$

Substituting (3) into (1) yields:

$$p_{z\max}(\Omega)=\frac{\beta}{\Omega}\left(\frac{\Omega\left(\beta-1\right)}{\beta}\right)^{\frac{\beta-1}{\beta}}\cdot\exp\left(-\frac{\beta-1}{\beta}\right) \tag{4}$$

From (4) we get:

$$\log\left(\max p_z\left(\Omega\right)\right)=k_1\log\Omega+n_1 \tag{5}$$

where the direction coefficient is:

$$k_1=-1/\beta \tag{6}$$

and the value on the ordinate axis is:

$$n_1=\left(1-\frac{1}{\beta}\right)\log\left(\beta-1\right)+\frac{1}{\beta}\log\beta-\frac{\beta-1}{\beta} \tag{7}$$

The envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the pdf is its particular solution:

$$p_z'(z)+p_z(z)\left(\frac{\beta}{\Omega}z^{\beta-1}-\frac{\beta-1}{z}\right)=0 \tag{8}$$

The graphic presentation of the dependence of the Weibull pdf versus parameter Ω, for fixed value of the fading parameter and different values of the received signal level, in logarithmic scale, is shown in Fig. 2.

The analysis of this dependence shows that with an increase of a received signal level the pdf maximums achieve lower values for higher values of parameter Ω, as shown in Fig. 2. It can also be concluded that all the maximums lie on a straight line, so the envelope of maximums is a straight line, on a logarithmic scale. In order to determine the values of the parameter Ω when the maximums are reached, and the values of maximums and the direction coefficient of the envelope, the first derivative of (1) relative to parameter Ω is determined.

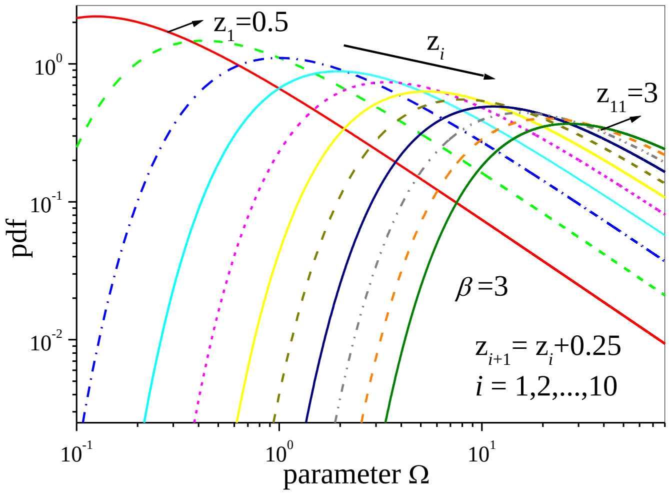

Figure 2. Weibull pdf versus Ω for different values of z and β=3

After equalization the first derivative of (1) relative to parameter Ω with zero it can be obtained:

$$\Omega_{p\max}=z^{\beta} \tag{9}$$

Substituting (9) into (1) yields:

$$p_{\Omega\max}(z)=\frac{\beta}{z}\exp(-1) \tag{10}$$

From (10) we get:

$$\log\left(\max p_z(z)\right)=k_2\log z+n_2 \tag{11}$$

where the direction coefficient is:

$$k_2=-1 \tag{12}$$

and the value on the ordinate axis is:

$$n_2=\log\beta-1 \tag{13}$$

The direction coefficient of the envelope has the value -1, regardless of the value of the fading depth parameter, while the value on the ordinate axis depends on this parameter. The envelope determines a certain singular solution of a differential equation which can describe the dynamics of this process, while the pdf is its particular solution:

$$p_z'(\Omega)+p_z(\Omega)\left(\frac{1}{\Omega}-\frac{z^{\beta}}{\Omega^2}\right)=0 \tag{14}$$

The previously described analysis is repeated for different values of the fading depth parameter, to discuss how this parameter affects the shape of the envelope of the pdf curves maximums, as it is presented in Fig. 3 and Fig. 4.

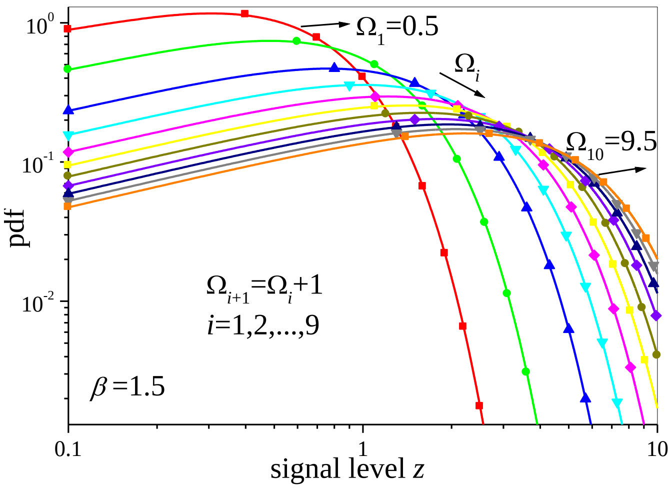

Figure 3. Weibull pdf versus z for different values of Ω and β=1.5

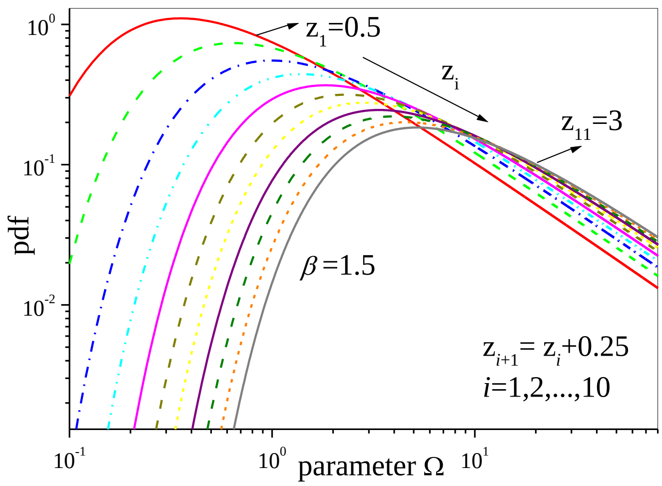

Figure 4. Weibull pdf versus Ω for different values of z and β=1.5

In the case β=1.5, the obtained results are presented in Fig. 3 and Fig. 4, respectively, showing the translation of the pdf curves family maximums envelope.

V. Results of Weibull pdf maximums position analysis

The envelopes of pdf curves family maximums for different values of fading depth parameter when the received signal level z is taken as a parameter, as well as in the case when the scale factor Ω is taken as a parameter, are graphically presented in Fig. 5 and Fig. 6., in logarithmic scale, using *Mathematica* numerical modeling.

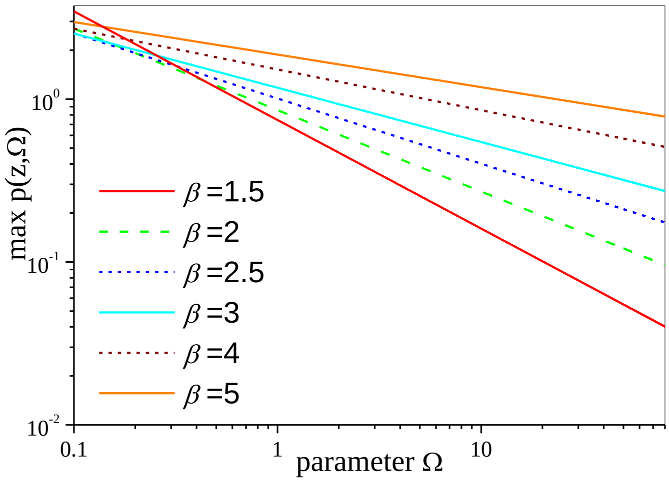

Figure 5. The envelopes of pdf curves family maximums for different values of fading depth parameter versus Ω

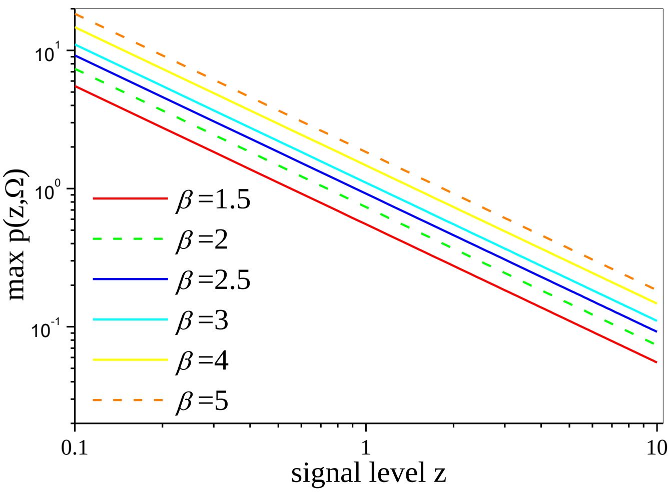

Figure 6. The envelopes of pdf curves family maximums for different values of fading depth parameter versus z

It can be concluded that an increase of fading depth parameter results in decrease (in absolute value) of envelope direction coefficient and increase of its value on the ordinate axis, when Weibull pdf curves maximums are analyzed versus parameter Ω, as it is presented in Fig. 5.

In the case when Weibull pdf curves maximums are analyzed versus parameter z, an increase of fading depth parameter results in increase of value on the ordinate axis, while the envelope direction coefficient has the same value, regardless of the value of fading depth parameter, as it is illustrated in Fig. 6.

Generating some data points using MATLAB, including the comparison with theoretical Weibull probability plot is given in Fig. 7 and Fig. 8, for different values of Weibull parameters. The dashed red line presents theoretical Weibull probability plot, while the data points are presented with different markers (+ or x).

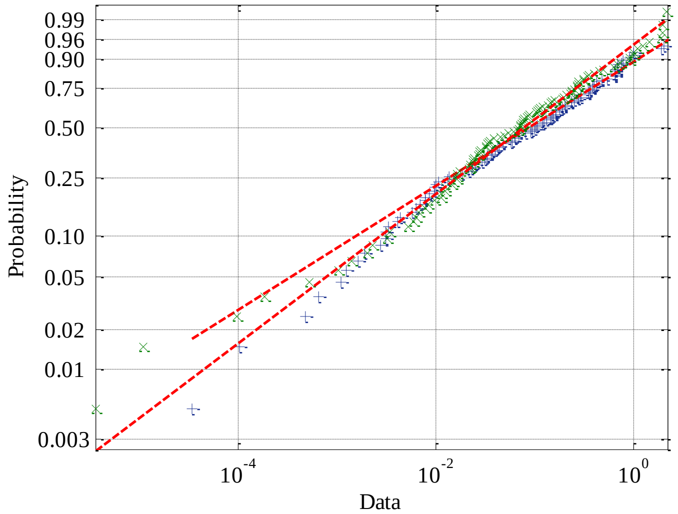

Figure 7. Generating data points and the comparison with theoretical probability plot for different values of parameter β

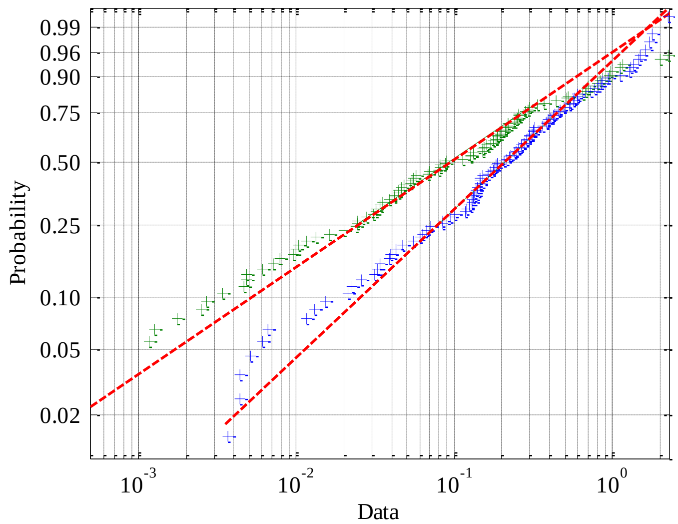

Figure 8. Generating data points and the comparison with theoretical probability plot for different values of parameter Ω

The assumption that the data comes from a Weibull distribution is reasonable, if the data points (presented with different markers in Fig. 7 and Fig. 8) fall near the red line.

VI. CONCLUSION

This paper presents some properties of the envelope of the pdf curves family in Weibull fading environment, including some numerical modeling and simulations. The obtained results show that the position of the maximums of these pdfs is uniquely determined by the maximums envelope's equation, which presents the singular solution of corresponding differential equation describing the complete dynamics of the signal transmission process.

In such a way, the boundary conditions for radio transmission, with given propagation conditions, could be defined, while some system performance measures could be evaluated, since they are all related to the received signal pdf.

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