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Convergence of Numerical Solutions for a Class of Stochastic Delay Age-dependent Capital System with Markovian Switching

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Abstract—**Recently, numerical solutions of stochastic differential equations have received a great deal of attention. Numerical approximation schemes are invaluable tools for exploring their properties. In this paper, we introduce a class of stochastic delay age-dependent (vintage) capital system with Markovian switching, and investigate the convergence of numerical approximation. The key aim is to show that the numerical solutions will converge to the true solutions under the local Lipschitz condition. A numerical example is provided to illustrate the theoretical results.**

Keywords- stochastic age-dependent capital syste; Markovian switching; delay; Euler approximation

I. Introduction

Over the last decade, systems with Marvokian jumps have been attracting increasing research attention [1,2,3,4]. The Markovian jump systems(MJSs) have the advantage of modelling the dynamic systems subject to abrupt variation in their structures, such as component failures or repairs, sudden environmental disturbance, changing subsystem interconnections, and operating in different points of a nonlinear plant[5].

Deterministic models of age-dependent (vintage) capital may be described by[6-8].

$$\begin{cases} \frac{\partial K(a,t)}{\partial t}+\frac{\partial K(a,t)}{\partial t} \\ =-\mu(a,t)K(a,t)+f(r(t),K(a,t)) & in \quad Q \\ K(0,t)=\phi(t)=\gamma(t)A(t)F(L(t),\int_0^A K(a,t)da) & in \quad t\in[0,T] \\ K(a,0)=K_0(0) & in \quad a\in(0,A) \\ N(t)=\int_0^A K(a,t)da & in \quad t\in[0,T] \end{cases} \quad (1)$$

where $Q=[0,A]\times[0,T]$, the stock of capital goods of age a at time t is denoted by $K(a,t)$, $N(t)$ is the total sum of the capital, a is the age of the capital, the investment t in the new capital,and the investment $f(t,K(a,t))$ in the capital of age a are the endogenous (unknown) variables. The maximum physical lifetime of capital A , the planning interval of calendar time $[0,T)$,the depreciation rate $\mu(a,t)$ of capital, and the capital density $K_0(a)$ (the initial distribution of capital over age) are given. $\gamma(t)$ denotes the accumulative rate at the moment of t; $0<\gamma(t)<1$, and $A(t)$ is the technical progress at the moment of t. Eq.(1) is a generalization of the deterministic capital equation. Eq.(1) describes the evolution of the composition of the productive capital as a function of purchasing/selling new or used capital. According to Eq.(1), machines of any age between 0 and A can be bought or sold.

The structure of $K(a,t)$ reflects different situations in economics and finance: its dependence on a illustrates the economic depreciation and physical deterioration of the capital, and represents the technological change embodied in capital. The case $K(a,t)/a>0$ corresponds to a technical progress when new capital is more efficient. In economics such models are known as vintage capital models (VCMs). They represent a new prospective mathematical tool for modeling technological innovation. It is a fast growing area of research. Its strong impact on mathematical finance is motivated by efficient description of fundamental finance characteristics such as cost of capital, risk of investment decisions, dynamics of finance investments, market uncertainty, etc. The validity of VCMs on real data is provided, i.e., in [9,10].

By Eq.(1), economy growth model focuses on four variables: output, capital, labor , and technological progress. Capital, labor, and technological progress are combined to produce output. However, some important sources of uncertainty may be discontinuous, recurrent, and fluctuating. Such significant events include innovations in technique, introduction of new products, natural disasters, and changes in laws or government policies. When capital, labor, and technological progress in corporates also abrupt changes in their structure, the Markovian jump system is very appropriate to describe their dynamics[4,11]. There exists an extensive literature dealing with stochastic differential equations with discontinuous paths incurred by L'evy processes (for instance, see monographs [12,13] and references therein). These equations are used as models in the study of queues, insurance risks, dams, and more recently in mathematical finance. In recent years, Markovian-switching models have attracted much attention by researchers and practitioners in economics and finance[14,15,16]. These models are able to incorporate the structural changes in the model dynamics, which might be attributed to the changes in macroeconomic conditions and different stages of business cycles. Now, applications of Markovian switching models can be found in various important fields in financial economics. Some of these applications include Elliott et al [14] for asset allocation, Elliott et al. [15] for short rate models, Elliott and Hinz [16] for portfolio analysis and chart analysis, Guo [17] and Buffington and Elliott[18] for option valuation, [15] for pricing and hedging volatility and variance swaps, and others. Recently, the spotlight has turned to the application of Markovian switching models to value options. Markovian switching models provide a more realistic way to describe the asset price dynamics for option valuation. They can incorporate the effect of structural changes in macro-economic conditions and business cycles on option valuation. In particular, the analytical pricing formula is given by the integral of the Black-Scholes-Merton formula and the occupation time of theregime-switching process. Guo [17] introduced a novel option pricing approach under a Markovian switching geometric Brownian motion (GBM).

Since time delay was first considered in the investment processes in [19], lots of literature such as have incorporated time lag into the dynamic economics and considered the impacts of delayed time on the whole economic system [20,21,22]. [20] analyzed an augmented IS-LM business cycle model with the capital accumulation equation that two time delays are considered in investment processes according to Kalecki's idea. Zak[21] investigated the Solow growth model with time lag, and considered that investment depended only on the capital stock at the past time and that the capital stock depreciated at the same gestation period, which it takes to produce and install capital goods.

We consider the following stochastic delay differential equations:

$$\begin{cases} \frac{\partial K(a,t)}{\partial t}+\frac{\partial K(a,t)}{\partial t} \\ =-\mu(a,t)K(a,t)+f(r(t),K(a,t),K(a,t-\tau)) \\ +g((r(t),K(a,t),K(a,t-\tau))\frac{dW_t}{dt}, & in \quad Q \\ K(0,t)=\phi(t)=\gamma(t)A(t)F(L(t),\int_0^A K(a,t)da) & in \quad t\in[0,T] \\ K(a,t)=\varphi(a,t), & in \quad \bar{R} \\ N(t)=\int_0^A K(a,t)da, & in \quad t\in[0,T] \\ r(0)=i_0, \end{cases} \qquad (2)$$

where $\overline{R}=[0,A]\times[-\tau,0]$. Uncertainty in the financial market is assumed to enter through the components of a Brownian motion W_t, and the components of a Markovian process.

Because, most stochastic modeling with Markovian switching are nonlinear and cannot have explicit solutions, so the construction of efficient computational methods is of great importance. For example, Yuan and Mao[23] gave the convergence of the Euler-Maruyama method for stochastic differential equations with Markovian switching, Li et. al[24] discussed the convergence of numerical solution to stochastic delay differential equation with Markovian switching, Zhou and Wu [25] investigated the convergence of numerical solutions to neutral stochastic delay differential equations with Markovian switching under the local Lipschitz condition.

However, to the best of our knowledge, there are not any numerical methods available for stochastic partial differential equations with Markovian switching. Inthis paper, we use the recent mathematical technique on the stochastic population system to estimate its numerical solutions . Some mathematical results may be found in [26,27,28]. We shall extend the idea from the papers [25, 29] to the numerical solutions for stochastic delay age-dependent capital system with Markovian switching. The main purpose of this paper is to investigate the convergence of numerical approximation of stochastic delay age-dependent capital system with Markovian switching under the local Lipschitz condition. In Section 2, we shall collect some basic preliminaries results which are essential for our development and the Euler approximation analysis, and Euler approximation is introduced. In Section 3, we give several lemmas which are useful for our main results. In Section 4, we shall show the main results that the numerical solutions will converge to the true solutions to stochastic delay age-dependent capital equations with Markovian switching under the given conditions. In section 5, A numerical example is provided to illustrate the theoretical results. Conclusion is given in section 6.

II. PRELIMINARIES AND APPROXIMATION

Throughout this paper, let

$$V=H^1([0,A])\equiv\Big\{\varphi\,|\,\varphi\in L^2([0,A]),\frac{\partial\varphi}{\partial x}\in L^2([0,A]),$$

$$where\ \frac{\partial\varphi}{\partial x}\ are\ \ generalized\ \ partial\ \ dervatives\Big\}.$$

V is a Sobolev space. $H=L^2([0,A])$ such that

$$V\rightarrow H\equiv H^{'}\rightarrow V^{'}.$$

Then $V'=H^{-1}([0,A])$ is the dual space of V. We denote by $|\cdot|$ and $\|\cdot\|$ the norms in V , and V' respectively; by $\langle\cdot,\cdot\rangle$ the duality product between V ,V', and by $(\cdot,\cdot)$ the scalar product in H. K is a real separable Hilbert space. For an operator $B \in L(K,H)$ being the space of all bounded linear operators from K into H ,we denote by $\|B\|_2$ the Hilbert-Scmidt norm,i.e.

$$\|B\|_2^2 = tr(BWB^T).$$

Let (Ω, F, P) be a complete probability space with a filtrations $\{F_t\}_{t\geq 0}$ satisfying the usual conditions(i.e.,it is increasing and right continuous while F_0 contains all p-null sets). $\tau > 0$ and $D : D([0,A]\times[-\tau,0];H)$ denotes the family of all right-continuous functions with left-hand limits φ from $[0,A]\times[-\tau,0]$ to H . The space $D([0,A]\times[-\tau,0];H)$ is assumed to be equipped with the norm $|\varphi|_D = \sup_{-\tau\leq x\leq 0}|\varphi(x)|.$ We also use $D^b_{F_0}([0,A]\times[-\tau,0];H)$ to denote the family of all almost surely bounded, F_0 -measurable, $D([0,A]\times[-\tau,0];H)$ -valued random variables.

Let $r(t), t > 0$,be a right-continuous Markov chain on the probability space taking values in a finite state $S = \{1,2,\cdots,N\}$ weth the generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t) = j \mid r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & if\ i \neq j; \\ 1+\gamma_{ij}\Delta + o(\Delta), & if\ i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while

$$\gamma_{ij} = -\sum_{i\neq j}\gamma_{ij}.$$

We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion W_t .It is well known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite subinterval of R^+ . Let $L^p_V = L^p([0,A]\times[0,T];V)$ and $L^p_H = L^p([0,A]$ $\times[0,T];H)$.

We consider stochastic delay age-dependent(vintage) capital system with Markovian switching (2), where $f : S\times L^2_H \times D([0,A])\times[-\tau,0]:H) \to H$ be a family of nonlinear operators, F_t -measurable almost surely in t . $g : S\times L^2_H \times D([0,A])\times[-\tau,0]:H) \to L(K,H)$ is the family of nonlinear operator, F_t -measurable almost surely in t . $K_0 = \varphi(a,0) \in D^b_{F_0}([0,A]\times[-\tau,0]:H).$ The integral version of Eq.(3) is given by the equation

$$\begin{aligned} K_t &= K_0 - \int_0^t \frac{\partial K_s}{\partial a}ds - \int_0^t \mu(a,s)K_s ds \\ &+ \int_0^t f(r(s),K_s,K_{t-\tau})ds + \int_0^t g(r(s),K_s,K_{t-\tau})dW_s \end{aligned} \tag{3}$$

here $K_t = K(a,t), K_{t-\tau} = K(a,t-\tau)$.

Give a stepsize $h \in (0,1)$, which satisfies $\tau = mh$ for some positive integer m , the discrete Markovian chain $r^h_k = r(kh), k = 0,1,2,\cdots,N$ can be simulated as follows: compute the one-step transition probability matrix $p(h)$ .Let $r^h_0 = i_0$ and generate a random number ξ_1 which is uniformly distributed in $[0,1]$. Define

$$r^h_k = \begin{cases} i, \quad if\ \in S-\{N\}\quad such\ that\ \sum_{j=1}^{i_1-1}P_{i0,j}(h) \leq \xi_1 < \sum_{i,j=1}^{i_1}p_{i0,j}(h), \\ N,\ if\ \ \sum_{j=1}^{N-1}Pi_0,j(h) \leq \xi_1 \end{cases}$$

where we set $\sum_{j=1}^{0}P_{i_0,j}(h) = 0$ as usual. generate independently a new random number ξ_2 which is again uniformly distributed in [0,1] and then define

$$r^h_k = \begin{cases} i_2 \quad if\quad i_2 \in S-\{N\}\ such\ that \\ \qquad\qquad \sum_{j=1}^{i_2-1}P_{i0,j}(h) \leq \xi_2 < \sum_{i,j=1}^{i_2}p_{r^h_1,j}(h), \\ N \quad if \quad \sum_{j=1}^{N-1}P_{r^h_1,j}(h) \leq \xi_2 \end{cases}$$

Repeating this procedure, a trajectory of $\{r^h_k, k = 1,2,\cdots N\}$ can be generated. This procedure can be carried out independently to obtain more trajectories. After explaining how to simulate the discrete Markov chain $\{r^h_k, k = 1,2,\cdots N\}$. we can now define Euler-Maruyama approximate solution for stochastic delay age-dependent (vintage) capital system with Markovian switching (2). For system (2) the discrete approximate solution on $t = 0, h, 2h, \cdots Nh = T$ is defined by the iterative scheme

$$Q_t^{k+1} = Q_t^k - \frac{\partial Q_t^{k+1}}{\partial a} h - \mu(a,t_k) Q_t^h h + f(r_k^h, Q_t^k, Q_t^{k-m}) h + g(r_k^h, Q_t^k, Q_t^{k-m}) \Delta W_t, \quad (4)$$

with initial value $Q_t^o \equiv K(a,0) = \varphi(a,0)$ on $\overline{R}$, $k > 1$ $r_k^h \equiv r(kh),$

$$Q^k(0,t) \equiv \gamma(t) A(t) F(L(t), \int_0^A Q_t^k da).$$

Here, Q_t^k is the approximation to $K(a,t_k)$, Q_t^{k-m} is the approximation to $K(a,t_k-\tau)$, for $t_k = kh$, the time increment is $h = T/N << 1$, Brownian motion increment is $\Delta W_k = W(t_{k+1}) - W_k$. We first define two step functions

$$z_1(t) = \sum_{k=0}^{N-1} Q_t^k 1_{[kh,(k+1)h)}(t),$$

$$z_2(t) = \sum_{k=0}^{N-1} Q_t^{k-m} 1_{[kh,(k+1)h)}(t), \quad (5)$$

$$\bar{r}(t) \equiv \sum_{k=0}^{N-1} r_k^h 1_{[kh,(k+1)h)}(t)$$

where 1_G is the indicator function for the set G . $t \in [0,T]$ then we define

$$K_t = K_0 - \int_0^t \frac{\partial Q_s}{\partial a} ds - \int_0^t \mu(a,s) Z_s ds + \int_0^t f(\bar{r}(s), Z_1(s), Z_2(s)) ds + \int_0^t g(\bar{t}, Z_1(s), Z_2(s)) dW_s. \quad (6)$$

we always assume that the following conditions aresatisfied:

(i) $\mu(a,t)$ is non-negative measurable in Q , $\gamma(t)$ and $A(t)$ are non-negative continuous in $[0,T]$ such that

$$\begin{cases} 0 \le \mu_0 \le \mu(a,t) \le \bar{\mu} \le \infty, \ in \quad Q, \\ Let \quad \gamma(t) \le \eta; \eta \ is \ a \ non-negative \ con\tan t; \end{cases}$$

where $\int_0^A \mu(a,t) da = +\infty$.

(ii) $f(i,0,0) = 0, \ \ g(i,0,0) = 0, \ i \in S;$

(iii) (local Lipschitz condition) for any bounded set $D \subseteq H$, there exists a positive constant $K_1(D)$ such that $x, y \in D, i \in S$

$$|f(i,x_1,y_1) - f(i,x_2,y_2)|^2 \vee \|g(i,x_1,y_1) - g(i,x_2,y_2)\|_2^2 \le K_1(D)(|x_1 - x_2|^2 + |y_1 - y_2|^2);$$

(iv)

$$\begin{cases} F(L,N) \ge 0, \ (F(L,0) = 0), \ \ \frac{\partial F}{\partial L} > 0, \\ 0 < \frac{\partial F}{\partial N} < F_1, \ where \ F_1 \ is \ a \ positive \ consant. \end{cases}$$

In an analogous way to the corresponding proof presented in [30], we may establish the following existence and uniqueness conclusion: under the conditions (i)-(iv), Eq.(2) has a unique continuous solution $K(a,t)$ on $(a,t) \in Q$.

III. Several Lemmas

we will need the following result. As for $r(t)$, the following lemma is satisfied (see [31]).

Lemma 1. Given $h > 0$, then $r_n^h = r(nh), n = 0,1,2,\cdots$ is a discrete Markovchain with the one-step transition probability matrix.

$$P(h) = (P_{ij}(h))_{N \times N} = e^{h\Gamma}.$$

Lemma 2. Under the conditions (i)-(iv), there are constants $k \ge 2$ and $C_1 > 0$ such that

$$E[\sup_{0 \le t \le T} |K_t|^k \le C_1.$$

Proof. Applying $It\hat{o}'s$ formula to $|K_t|^k$, we obtain

$$\begin{aligned} &|K_t|^k \\ &= |K_0|^k + \int_0^t k|K_s|^{k-2} < -\frac{\partial K_s}{\partial a} - \mu(a,s) K_s, K_s > ds \\ &+ k\int_0^t |K_s|^{k-2} (f(r(s), K_s, K_{s-\tau}), K_s) ds \\ &+ k\int_0^t |K_s|^{k-2} (g(r(s), K_s, K_{s-\tau}), K_s) dW_s \\ &+ \frac{k(k-2)}{2} \int_0^t |K_s|^{k-4} \|(K_s, g(r(s), K_s, K_{s-\tau})) dW_s\|_2^2 \\ &+ \frac{k}{2} \int_0^t |K_s|^{k-2} \|g(r(s), K_s, K_{s-\tau})\|_2^2 ds \end{aligned}$$

$$\leq |K_0|^k - \int_0^t k|K_s|^{k-2} < \frac{\partial K_s}{\partial a}, K_s > ds$$
$$- k\mu_0 \int_0^t |K_s|^{k-2} (K_s, K_s) ds$$
$$+ k\int_0^t |K_s|^{k-2} (f(r(s), K_s, K_{s-\tau}), K_s) ds$$
$$+ k\int_0^t |K_s|^{k-2} (g(r(s), K_s, K_{s-\tau}), K_s) dW_s$$
$$+ \frac{k(k-2)}{2} \int_0^t |K_s|^{k-4} \|(K_s, g(r(s), K_s, K_{s-\tau})) dW_s\|_2^2$$
$$+ \frac{k}{2} \int_0^t |K_s|^{k-2} \|g(r(s), K_s, K_{s-\tau})\|_2^2 ds$$

Since

$$- < \frac{\partial K_s}{\partial a}, K_s > = -\int_0^t K_s d_a(K_s)$$
$$= \frac{1}{2}\gamma^2(s)A^2(s)[F(L(s), \int_0^A K_s da) - F(L(s), 0)]^2$$
$$\leq \frac{1}{2}\eta^2 (\frac{\partial F(L,N)}{\partial N}|_y)^2 (\int_0^A K_s da) \leq \frac{1}{2} A F_1^2 \eta^2 |K_s|^2$$

where $y \in (0, \int_0^A K_s da)$.

Therefore, by conditions (i) and (iii), we get that

$$|K_t|^k \leq |K_0|^k + \frac{k}{2}(AF_1^2\eta^2 - 2\mu_0)\int_0^t |K_s|^k ds$$
$$+ k\int_0^t |K_s|^{k-1} |f(r(s), K_s, K_{s-\tau}), K_s| ds$$
$$+ k\int_0^t |K_s|^{k-2} (g(r(s), K_s, K_{s-\tau}), K_s) dW_s$$
$$+ \frac{k(k-1)}{2} \int_0^t |K_s|^{k-2} \|g(r(s), K_s, K_{s-\tau}))\|_2^2 ds$$
$$\leq |K_0|^k + \frac{k}{2}(AF_1^2\eta^2 - 2\mu_0)\int_0^t |K_s|^k ds$$
$$+ kK_1(D)\int_0^t |K_s|^K ds + kK_1(D)\int_0^t |K_s|^{k-1} |K_{s-\tau}| ds$$
$$+ k\int_0^t |K_s|^{k-2} (g(r(s), K_s, K_{s-\tau}), K_s) dW_s$$
$$+ \frac{k(k-1)}{2} K_1(D) \int_0^t |K_s|^{k-2} (|K_s|^2 + |K_{s-\tau}|^2) ds$$

Now, it follows that for any $t \in [0, T]$

$$E(\sup_{0\leq t\leq T} |K_t|^k)$$
$$\leq E|K_0|^k + \frac{k}{2}(AF_1^2\eta^2 - 2\mu_0 + 2kK_1(D)$$
$$+ 4K_1(D))\int_0^t E \sup_{-\tau\leq t\leq T} |K_s|^k ds \qquad (7)$$
$$+ kE \sup_{-\tau\leq s\leq T} \int_0^s |K_u|^{k-2} (g(r(u), K_u, K_{u-\tau}), K_u) dW_u$$

Whence applying the Burkholder-Davis-Gundy's inequality to the last term of (7) leads to

$$E[\sup_{0\leq s\leq T} \int_0^s |K_u|^{k-2} (g(r(u), K_u, K_{u-\tau}), K_u) dW_u]$$
$$\leq 3E[\sup_{0\leq s\leq t} |K_s|^{k/2} (\int_0^t (|K_s|^{k-2} \|g(r(s), K_s, K_{u-\tau})\|_2^2 ds)^{1/2}]$$
$$\leq \frac{1}{2k} E[\sup_{0\leq s\leq t} |K_s|^k] + 2K_0K_1(D)\int E[\sup_{-\tau\leq s\leq t} |K_s|^k ds] \qquad (8)$$

where K_0 is a constant. Thus, it follows from (7) and (8)

$$E(\sup_{0\leq t\leq T} |K_t|^k)$$
$$\leq 2E|K_0|^k + k(AF_1^2\eta^2 - 2\mu_0 + 2kK_1(D) + 4K_1(D)$$
$$+ 4kK_0K_1(D))\int_0^T E \sup_{-\tau\leq r\leq s} |K_r|^k ds, \quad \forall t \in [0, T].$$

Now, Gronwall's lemm a obviously implies the required result with

$$C_1 = 2e^{kT|AF_1^2\eta^2 - 2\mu_0 + 2kK_1(D) + 4K_1(D) + 4kK_0K_1(D)|} E|K_0|^k.$$

Lemma 3. Under the conditions (i)-(iv), there is a constant $C_2 > 0$ such that

$$E\left[\sup_{-\tau\leq t\leq T} |Q_{t\wedge\tau}|^2\right] \leq C_2,$$

where τ is the stopping times defined by $\sigma = \rho \wedge \theta$ $\rho = \inf\{t \geq 0 : Q_t \,\overline{\in}\, D\}, and\ \theta = \inf\{t \geq 0 : K_t \,\overline{\in}\, D\}$ are the first time that Q_t and K_t leave a bounded region D respectively . We will define D more precisely later.

Proof. From (6), one can obtain

$$dQ_t = -\frac{\partial Q_t}{\partial a}dt - \mu(a,t)z_1(t)dt + f(\bar{r}(t), z_1(t), z_2(t))dt + g(\bar{r}(t), z_1(t), z_2(t))dW_t \quad (9)$$

Applying $Itô's$ formula to $|Q_t|^2$ yields

$$|Q_{t\wedge\sigma}|^2 = |Q_0|^2 + 2\int_0^{t\wedge\sigma} < -\frac{\partial Q_t}{\partial a}dt - \mu(a,t)z_1(t), Q_s > ds + 2\int_0^{t\wedge\sigma}(f(\bar{r}(t), z_1(t), z_2(t)), Q_s)ds + 2\int_0^{t\wedge\sigma}(g(\bar{r}(t), z_1(t), z_2(t)), Q_s)dW_s + 2\int_0^{t\wedge\sigma}\| g(\bar{r}(s), z_1(t), z_2(t))\|_2^2\, ds$$

Therefore, we get that

$$\begin{aligned}|Q_{t\wedge\sigma}|^2 \le |Q_0|^2 &+ \frac{1}{2}AF_1^2\eta^2\int_0^{t\wedge\sigma}|Q_s|ds \\ &+ \int_0^{t\wedge\sigma}| f(\bar{r}(s), z_1(t), z_2(t))|^2\, ds \\ &+ 2\bar{\mu}\int_0^{t\wedge\sigma}|Q_s||z_1(s)|ds + \int_o^{t\wedge\sigma}|Q_s|^2 ds \\ \vdots \quad &+ \int_0^{t\wedge\sigma}\| g(\bar{r}(s), z_1(t), z_2(t))\|_2^2\, ds \\ &+ 2\int_0^{t\wedge\sigma}(Q_s, g(\bar{r}(s), z_1(t), z_2(t)))dW_s\end{aligned}$$

Now, it follows that for any $t \in [0,T]$

$$\begin{aligned}&E(\sup_{0\le t\le T}|Q_{t\wedge\sigma}|^2) \\ &\le E|Q_0|^2 + (\frac{1}{2}AF_1^2\eta^2 + 2\bar{\mu} + 1)E\sup_{0\le t\le T}\int_o^{t\wedge\sigma}|Q_s|^2 ds \\ &+ E\sup_{0\le t\prec T}\int_0^{t\wedge\sigma}E|f(\bar{r}(s), z_1(s), z_2(s))|^2 ds \\ &+ E\sup_{0\le t\prec T}\int_0^{t\wedge\sigma}E\|g(\bar{r}(s), z_1(s), z_2(s))\|_2^2 ds \\ &+ 2E\sup_{0\le t\le T}\int_0^{t\wedge\sigma}(Q_s, g(\bar{r}(s), z_1(s), z_2(s)))dW_s.\end{aligned}$$

Using condition (iii) yields

$$\begin{aligned}&E(\sup_{0\le t\le T}|Q_{t\wedge\sigma}|^2) \\ &\le E|Q_0|^2 + (\frac{1}{2}AF_1^2\eta^2 + 2\bar{\mu} + 1)E\sup_{0\le t\le T}\int_o^{t\wedge\sigma}|Q_s|^2 ds \\ &+ 2K_1(D)E\sup_{0\le t\le T}\int_0^{t\wedge\sigma}(|z_1(s)|^2 + |z_2(s)|^2)ds \\ &+ 2E\sup_{0\le t\le T}\int_0^{t\wedge\sigma}(Q_s, g(\bar{r}(s), z_1(s), z_2(s)))dW_s.\end{aligned} \quad (10)$$

By Burkholder-Davis-Gundy's inequality , we have

$$\begin{aligned}&E[\sup_{0\le t\le T}\int_0^{t\wedge\sigma}(Q_s, g(\bar{r}(s), z_1(s), z_2(s)))dW_s \\ &\le 3E[\sup_{0\le t\le T}|Q_{t\wedge\sigma}|(\int_0^{t\wedge\sigma}\|g(\bar{r}(s), z_1(s), z_2(s)\|_2^2\, ds)^{1/2}] \\ &\le \frac{1}{4}E[\sup_{0\le t\le T}|Q_{t\wedge\sigma}|^2] \\ &+ K_1\int_0^{t\wedge\sigma}\|g(\bar{r}(s), z_1(s), z_2(s)\|_2^2\, ds) \\ &\le \frac{1}{4}E[\sup_{0\le t\le T}|Q_{t\wedge\sigma}|^2] \\ &+ K_1\cdot K_1(D)\int_0^{t\wedge\sigma}E(|z_1(s)|^2 + |z_2(s)|^2)ds \\ &\le \frac{1}{4}E[\sup_{0\le t\le T}|Q_{t\wedge\sigma}|^2] \\ &+ K_1\cdot K_1(D)\int_0^T E(\sup_{-\tau\le u\le t\wedge\sigma}|Q(s)|^2)ds.\end{aligned} \quad (11)$$

For some positive constant $K_1 > 0$. For $\forall t \in [0,T]$, it follows from (10) and (11)

$$\begin{aligned}&E\sup_{-\tau\le t\le T}|Q_{t\wedge\sigma}|^2 \\ &\le 2E|Q_0|^2 + 2(AF_1^2\eta^2 + 2\bar{\mu} + 2K_1(D) \\ &+ 2K_1K_1(D) + 1)\int_0^T E\sup_{-\tau\le t\le s}|Q_{r\wedge\sigma}|^2 ds\end{aligned}$$

Applying Gronwall's lemma, $\forall t \in [0,T]$ one can get

$$E\sup_{-\tau\le t\le T}|Q_{t\wedge\sigma}|^2 \le 2e^{2(AF_1^2\eta^2+2\bar{\mu}+2K_1(D)+2K_1K_1(D)+1)}E|Q_0|^2=C_2.$$

The proof is finished.

Lemma 4. For any $t\in[0,T]$, there are constants $C_3>0$ and $C_4>0$ such that

$$E\int_0^t|f(\bar{r}(s),z_1(s),z_2(s))-f(r(s),z_1(s),z_2(s))|^2\,ds \le C_3h+o(h);$$

$$E\int_0^t\|g(\bar{r}(s),z_1(s),z_2(s))-f(r(s),z_1(s),z_2(s))\|_2^2\,ds \le C_4h+o(h).$$

The proof is similar to that in [25].

Lemma 5. Under the conditions (i)-(iii), there are constants $C_5>0$ and C_6 such that

$$E\sup_{0\le t\le T}|Q_t-z_1(t)|^2\le C_5h; \quad (12)$$

$$E\sup_{0\le t\le T}|Q_{t-\tau}-z_2(t)|^2\le C_6h. \quad (13)$$

The proof of this analogous to that in [24]

IV. Main results

Now we are in position to establish the following main results.

Theorem 6. Under the assumptions (i)-(iv), then

$$E\sup_{0\le t\le T}|K_t-Q_t|^2\le C_7(D)h+o(h). \quad (14)$$

Proof. By the definitions of K_t and Q_t, we have

$$\begin{aligned}&K_t-Q_t\\&=\int_0^t\frac{\partial(K_s-Q_s)}{\partial a}ds-\int_0^t\mu(a,s)(K_s-z_1(s))ds\\&+\int_0^t(f(r(s),K_s,K_{s-\tau})-f(\bar{r}(s),z_1(s),z_2(s)))ds\\&+\int_0^t(g(r(s),K_s,K_{s-\tau})-g(\bar{r}(s),z_1(s),z_2(s)))dW_s.\end{aligned}$$

Therefore using $It\hat{o}'s$ formula, along withthe Cauchy-Schwarz's inequality yields,

$$\begin{aligned}&d|K_t-Q_t|^2\\&=2\left\langle K_t-Q_t,\frac{\partial(K_s-Q_s)}{\partial a}\right\rangle dt\\&-2\langle K_t-Q_t,\mu(a,t)(K_t-z_1(s))dt\rangle\\&+2(K_t-Q_t,f(r(t),K_t,K_{s-\tau})-f(\bar{r}(t),z_1(t),z_2(t)))dt\\&+\|g(r(t),K_t,K_{s-\tau})-g(\bar{r}(t),z_1(t),z_2(t))\|_2^2\,dt+\\&2(K_t-Q_t,(g(r(t),K_t,K_{s-\tau})\\&-g(\bar{r}(t),z_1(t),z_2(t)))dW_t)\end{aligned}$$

for any $t_1\in[0,T]$,

$$\begin{aligned}&E\sup_{s\in[0,t_1]}|K_{s\wedge\sigma}-Q_{s\wedge\sigma}|^2\\&\le(AF_1^2\eta^2+\bar{\mu}+1+2K^2)\int_0^{t\wedge\sigma}E\sup_{t\in[0,t_1]}|K_s-Q_s|^2\\&+2\bar{\mu}E\int_0^{t\wedge\sigma}E\sup_{t\in[0,t_1]}|K_s-z_1(s)|^2ds\\&+E\sup_{s\in[0,t_1]}\left|\int_0^{t\wedge\sigma}(f(r(s),K_s,K_{s-\tau})\right.\\&\left.-f(\bar{r}(s),z_1(s),z_2(s)))ds\right|^2\\&+E\sup_{s\in[0,t_1]}\int_0^{t\wedge\sigma}\|g(r(s),K_s,K_{s-\tau})\\&-g(\bar{r}(s),z_1(s),z_2(s))\|_2^2\,ds\\&+2E\sup_{s\in[0,t_1]}\int_0^{t\wedge\sigma}(K_s-Q_s,(g(r(s),K_s,K_{s-\tau})\\&-g(\bar{r}(s),z_1(s),z_2(s)))dsW_s).\end{aligned} \quad (15)$$

By Lemma 4, Lemma 5 and condition (iii), one gets

$$\begin{aligned}&E\sup_{t\in[0,t_1]}\int_0^{t\wedge\sigma}|(f(r(s),K_s,K_{s-\tau})\\&-f(\bar{r}(s),z_1(s),z_2(s))|^2\,ds\\&\le 2E\int_0^{t\wedge\sigma}|(f(r(s),K_s,K_{s-\tau})\\&-f(r(s),z_1(s),z_2(s))|^2\,ds\end{aligned}$$

$$+2E\int_0^{t\wedge\sigma}|(f(r(s),z_1(s),z_2(s))$$
$$-f(\bar{r}(s),z_1(s),z_2(s))|^2\ ds$$
$$\le 2K_1(D)E\int_0^{t\wedge\sigma}(|K_s-z_1(s)|^2+|K_{s-\tau}-z_2(s)|^2)ds$$
$$+2E\int_0^{t\wedge\sigma}|(f(r(s),z_1(s),z_2(s))$$
$$-f(\bar{r}(s),z_1(s),z_2(s))|^2\ ds$$
$$\le 4K_1(D)E\int_0^T E[\sup_{u\in[0,s]}|K_{u\wedge\sigma}-Q_{u\wedge\sigma}|^2\ ds$$
$$+2K_1(D)(C_5+C_6)h+2C_3h+o(h). \tag{16}$$

Similarly

$$E\sup_{t\in[0,t_1]}\int_0^{t\wedge\tau}\|(g(r(s),K_s,K_{s-\tau})$$
$$-g(\bar{r}(s),z_1(s),z_2(s))\|_2^2\ ds \tag{17}$$
$$\le 4K_1(D)E\int_0^T E[\sup_{u\in[0,s]}|K_{u\wedge\rho}-Q_{u\wedge\rho}|^2\ ds$$
$$+2K_1(D)(C_5+C_6)h+2C_4h+o(h).$$

By Burkholder-Davis-Gundy's inequality, we have

$$E\sup_{t\in[0,t_1]}\int_0^{t\wedge r}(K_s-Q_s,(g(r(s),K_s,K_{s-\tau})$$
$$-g(\bar{r}(s),z_1(s),z_2(s)))dsW_s).$$
$$\le\frac{1}{4}E[\sup_{0\le t\le t_1}|K_t-Q_t|^2] \tag{18}$$
$$+4K_2K_1(D)\int_0^T E[\sup_{0\le t\le t_1}|K_{u\wedge\rho}-Q_{u\wedge\rho}|^2\ ds$$
$$+2k_2K_1(D)(C_5+C_6)h+2K_2C_4h+o(h)$$

where K_1 and K_2 are positive constants.
Therefore inserting (16)-(18) into (15) has

$$E\sup_{t\in[0,T]}|K_t-Q_t|\le M_1\int_0^T E\sup_{r\in[0,s]}|K_r-Q_r|^2ds$$
$$+M_2h+o(h)+\frac{1}{2}E\sup_{r\in[0,T]}|K_t-Q_t|^2,$$

where

$$M_1=$$
$$(AF_1^2\eta^2+3\bar{\mu}+1+2K^2+8K_1(D)+8k_2K_1(D)),.$$

$$M_2=2(\mu C_5+C_3+C_4+2K_1(D)(C_5+C_6)$$
$$+2k_2K_1(D)(C_5+C_6)+K_2C_4).$$

On applying Gronwall inequality we can obtain a bound of the from

$$E\sup_{0\le t\le T}|K_t-Q_t|^2\le 2M_2e^{2M_1}h+o(h)$$

The result then follows with $C_6=2M_2e^{2M_1}$.
It is easy to deduce that the following theorem is satisfied. To proceed further we define the bounded domain

$$D=D(r)\equiv\{y\in G\ such\quad that\quad |y|^2\le r\}$$

It is easy to deduce that the following theorem is satisfied.
Theorem 7. Under the assumptions (i)-(iv), the numerical approximate solution (6) will converge to the exact solution to Eq.(2) in the sense then

$$\lim_{\Delta t\to 0}E\left[\sup_{0\le t\le T}|K_t-Q_t|^2\right]=0 \tag{19}$$

Theorem 8. If θ is the first exist time of the solution K_t to equation (3) from the domain $D(r)$,then the probability

$$P(\theta\ge T)\ge 1-\varepsilon.$$

Proof Whence applying Lemma 2 leads to

$$E\left[\sup_{0\le t\le T}|K_t|^2\right]\le C_1.$$

On noting that $|K_\theta|^2=r$, since K_θ is on the boundary of $D(r)$, the probability $P(\theta<T)$ can now be bounded as follows.

$$C_1\ge E[|K_{t\wedge\theta}|^2]\ge E[|K_{t\wedge\theta}|^2 I_{\{\theta<t\}}(\varpi)]$$
$$\ge rE[I_{\{\theta<t\}}(\varpi)]\ge rP(\theta<T) \tag{20}$$

whence rearranging (20) leads to

$$P(\theta < T) \le C_1 / r = \varepsilon . \quad (21)$$

Here r can be made as large as required, for a given T and K_0, to accommodate any $\varepsilon \in (0,1)$. Theorem 7 is proved.

We note that the following useful result follows directly from Theorem 8.

Lemma 9. let θ be the first exist time of the solution K_t to equation (2) from the domain $D(r)$, and let conditions (i)-(iv) are satisfied, then the limit of $\lim_{r\to\infty} D(r) \equiv G$ and, for $t \in [0,T]$ and $K_0 \in G$, K_t remains in G . Furthermore, K_t is the unique solution of equation (2) on $t \in [0,T]$ for all finite T .
Proof of this result can be found in paper of Mao[32].

We require a similar result to Theorem 10 for the Euler approximate solution Q_t .

Theorem 10. let ρ be the first exit time of the Euler approximate solution (6) from the domain $D(r)$. Suppose conditions (i)-(iv) are satisfied, then (for sufficiently small h) probability

$$P(\rho \ge T) \ge 1 - \varepsilon M,$$

where M is a constant.

Proof Noting that Q_t is the solution to (6), and apply Lemma 3 leads to

$$E[|Q_{\rho\wedge t}|^2] \le C_2 .$$

An argument analogous to that used to prove Theorem 2 can now be used to bound $P(\rho < T)$ .Since Q_ρ is on the boundary of $D(r)$ then $|Q_\rho|^2 = r$ which leads to

$$C_2 \ge rE[I_{\{\rho<T\}}(\varpi)] \ge rP(\rho < T).$$

Rearranging this inequality reveals that

$$P(\rho < T) \le M\varepsilon,$$

where $M = C_2 / C_1$, ε is defined in equation (21).

The significance of Theorem 8 and Theorem 10 are that both K_t and Q_t remain within the domain $D(r)$ and therefore by Theorem 7 the Euler scheme will converge to the K_t, with probability

$$P(\sigma < T) \le P(\rho < T) + P(\theta < T) \le (1+M)\varepsilon. \quad (22)$$

Theorem 11 . let G be an open subset of H , and denote the unique solution of (2) for $t \in [0,T]$ given $K_0 \in G$ by $K_t \in G$. Define Q_t as the Euler approximation (6) and let $D \subseteq G$ be any bounded set. Suppose conditions (i)-(iv) are satisfied. Then for any ε , $\delta > 0$ there exists $\Delta t^* > 0$ such that

$$P(\sup_{0\le t\le T} |Q_t - K_t|^2 \ge \delta) \le \varepsilon,$$

provided $h \le \Delta t^*$ and the initial value $K_0 \in G$.
Proof of this result can be found in paper of Mao[32].

V. An Example

In this section we shall discuss an example to illustrate our theory.

Example**.** Let W_t be a scalar Brownian motion. Let $r(t)$ be a right continuous Markovian chain taking values in $S = \{1,2\}$ with the generator

$$\Gamma = (\gamma_{ij})_{2\times 2} = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}.$$

Of course W_t and $r(t)$ are assumed to be independent. Let us consider a stochastic delay capital system of the form

$$\begin{cases} \dfrac{\partial K(a,t)}{\partial t} + \dfrac{\partial K(a,t)}{\partial a} \\ = -2\dfrac{1}{(1-a)^2} K(a,t) \\ + f(r(t), K(a,t), K(a,t-\tau))dt \\ + g(r(t), K(a,t), K(a,t-\tau))dW_t \\ K(a,t) = \exp(-\dfrac{t+1}{1-a}), \\ K(0,t) = \gamma(t)A(t)\dfrac{2}{(1-t)^2}\int_0^1 K(a,t)da \\ r(0) = 1 \end{cases} \quad (23)$$

Here $T=1$, $\tau=0.1$, $(a,t)\in(0,1)\times(0,T)$, $(a,t)\in(0,1)\times[-0.1,0]$, $H=L^2([0,1])$, $V=W_0^1([0,1])$ (a Sobolev space with elements satisfying the boundary conditions above),

$$\mu(a,t)=2\frac{1}{(1-a)^2},\ L(t)=\frac{2}{(1-t)^2}\quad \lambda(t)A(t)=2,$$
$$F(L(t),\int_0^1 K(a,t)da)=\frac{2}{(1-t)^2}\int_0^1 K(a,t)da,$$
$$f(1,K(a,t),K(a,t-\tau))=-K(a,t)+K(a,t-\tau)),$$
$$f(2,K(a,t),K(a,t-\tau))=\cos(K(a,t))+K^2(a,t-\tau)$$

and

$$g(1,K(a,t),K(a,t-\tau))=\sin(K(a,t))+2K(a,t-\tau),$$
$$g(2,K(a,t),K(a,t-\tau))=K^2(a,t)+2K(a,t-\tau),$$
$$K_0(a)=\exp(-\frac{1}{1-a}).$$

It is easy to verify that the conditions (i)-{(iv) are satisfied. Then, the approximate solution will converge to the true solution of (23) for any $(a,t)\in(0,1)\times(0,t)$ in the sense of Theorem 10.

Obviously, $K(a,t)$ in (23) cannot be solved explicitly. It is necessary to know the numerical approximation $Q(a,t)$ of $K(a,t)$. Take $h=0.0005$, $\Delta a=0.05$. Fig.1 is numerical simulations of the stochastic age-dependent capital system with Markovian switching (24) with 1000 experiments, where

$$K(a,t)=EQ(a,t)=\frac{1}{1000}\sum_{k=1}^{1000}Q_K(a,t).$$

It clearly reveals the age-dependent capital system tendency.

VI. Conclusion

Some important sources of uncertainty may be discontinuous, recurrent, and fluctuating. Such significant events include innovations in technique, introduction of new products, natural disasters, and changes in laws or government policies. The relationship among these events and the profitability of risky assets can be very complicated. Furthermore, there can be numerous events and economic variables that are potentially related to the profitability of risky assets. In order to describe this situation, we introduce a class of stochastic age-dependent capital dynamic system. To the best of our knowledge, there are not any numerical methods available for stochastic partial differential equations with Markovian switching. Thus, numerical approximation schemes are invaluable tools for exploring its properties.

In this paper, we extend the idea from the papers [25,29] to the numerical solutions for stochastic delay age-dependent capital system with Markovian switching. Using the recent mathematical technique for the stochastic differential equations, this paper investigates the convergence of numerical approximation of stochastic delay age-dependent capital system with Markovian switching under the given conditions. The paper obtains the condition that can ensure that the approximate solution will converge to the true solution for stochastic delay age-dependent capital system. At the same time, we propose the numerical solution for stochastic delay age-dependent capital system with Markovian switching. The approach is based on constructing a discrete-time approximation to exact solution by consider the jump time. An example has been given for illustration of our theory.

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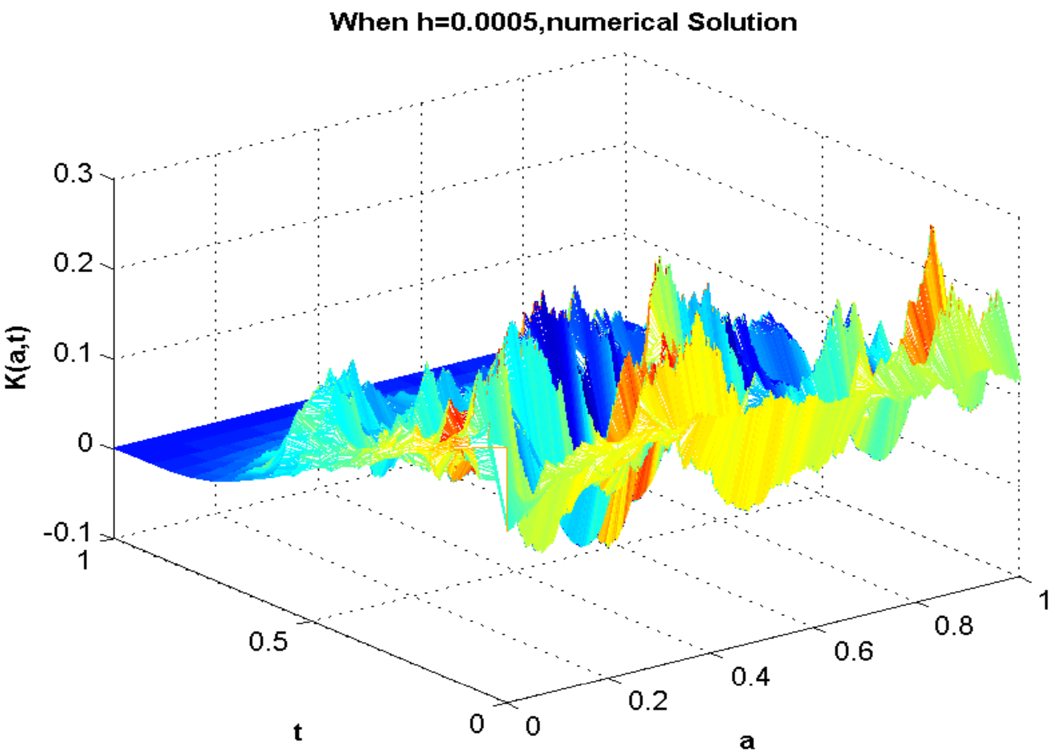

Fig.1 numerical simulations of the stochastic age-dependent capital system with Markovian switching